

Frank W. Warner

FOUNDATIONS  
of DIFFERENTIABLE MANIFOLDS  
and LIE GROUPS

With 57 Illustrations



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# Contents

## 1 MANIFOLDS

- 2 Preliminaries
- 5 Differentiable Manifolds
- 8 The Second Axiom of Countability
- 11 Tangent Vectors and Differentials
- 22 Submanifolds, Diffeomorphisms, and the Inverse Function Theorem
- 30 Implicit Function Theorems
- 34 Vector Fields
- 41 Distributions and the Frobenius Theorem
- 50 Exercises

## 2 TENSORS AND DIFFERENTIAL FORMS

- 54 Tensor and Exterior Algebras
- 62 Tensor Fields and Differential Forms
- 69 The Lie Derivative
- 73 Differential Ideals
- 77 Exercises

## 3 LIE GROUPS

- 82 Lie Groups and Their Lie Algebras
- 89 Homomorphisms
- 92 Lie Subgroups
- 98 Coverings
- 101 Simply Connected Lie Groups
- 102 Exponential Map
- 109 Continuous Homomorphisms
- 110 Closed Subgroups
- 112 The Adjoint Representation
- 117 Automorphisms and Derivations of Bilinear Operations and Forms
- 120 Homogeneous Manifolds
- 132 Exercises

**4 INTEGRATION ON MANIFOLDS**

- 138 Orientation
- 140 Integration on Manifolds
- 153 de Rham Cohomology
- 157 Exercises

**5 SHEAVES, COHOMOLOGY, AND THE DE RHAM THEOREM**

- 163 Sheaves and Presheaves
- 173 Cochain Complexes
- 176 Axiomatic Sheaf Cohomology
- The Classical Cohomology Theories
- 186 *Alexander-Spanier Cohomology*
- 189 *de Rham Cohomology*
- 191 *Singular Cohomology*
- 200 *Čech Cohomology*
- 205 The de Rham Theorem
- 207 Multiplicative Structure
- 214 Supports
- 216 Exercises

**6 THE HODGE THEOREM**

- 220 The Laplace-Beltrami Operator
- 222 The Hodge Theorem
- 227 Some Calculus
- 240 Elliptic Operators
- 243 Reduction to the Periodic Case
- 250 Ellipticity of the Laplace-Beltrami Operator
- 251 Exercises

**260 BIBLIOGRAPHY**

**262 SUPPLEMENT TO THE BIBLIOGRAPHY**

**264 INDEX OF NOTATION**

**267 INDEX**