GEOMETRY AND ALGEBRA IN ANCIENT CIVILIZATIONS

B. L. van der Waerden

Geometry and Algebra in Ancient Civilizations

With 98 Figures

Springer-Verlag
Berlin Heidelberg New York Tokyo
1983

Prof. Dr. B. L. van der Waerden
Mathematisches Institut der Universität Zürich

Cover illustration

One of Tai Chen's illustrations of the "Nine Chapters of the Mathematical Art", explaining Liu Hui's method of measuring the circle (see pages 196–199). Reproduced from Joseph Needham: Science and Civilization in China, Volume 3, p. 29.

ISBN-13:978-3-642-61781-2 e-ISBN-13:978-3-642-61779-9
DOI: 10.1007/978-3-642-61779-9

Library of Congress Cataloging in Publication Data
Waerden, B. L. (Bartel Leenert) van der, 1903–
Geometry and algebra in ancient civilizations.
Includes index.
Contents:
1. Algebra - History. I. Title.
QA151.W34. 1983. 512'.009. 83-501
ISBN-13:978-3-642-61781-2 (U.S.)

Softcover reprint of the hardcover 1st edition 1983

2141/3140-543210

Preface

Originally, my intention was to write a "History of Algebra", in two or three volumes. In preparing the first volume I saw that in ancient civilizations geometry and algebra cannot well be separated: more and more sections on ancient geometry were added. Hence the new title of the book: "Geometry and Algebra in Ancient Civilizations". A subsequent volume on the history of modern algebra is in preparation. It will deal mainly with field theory, Galois theory and theory of groups.

I want to express my deeply felt gratitude to all those who helped me in shaping this volume. In particular, I want to thank Donald Blackmore Wagner (Berkeley) who put at my disposal his English translation of the most interesting parts of the Chinese "Nine Chapters of the Art of Arithmetic" and of Liu Hui's commentary to this classic, and also Jacques Sesiano (Geneva), who kindly allowed me to use his translation of the recently discovered Arabic text of four books of Diophantos not extant in Greek. Warm thanks are also due to Wyllis Bandler (Colchester, England) who read my English text very carefully and suggested several improvements, and to Annemarie Fellmann (Frankfurt) and Erwin Neuenschwander (Zürich) who helped me in correcting the proof sheets. Miss Fellmann also typed the manuscript and drew the figures.

I also want to thank the editorial staff and production department of Springer-Verlag for their nice cooperation.

Zürich, February, 1983 B. L. van der Waerden

Table of Contents

Introduction

Until quite recently, we all thought that the history of mathematics begins with Babylonian and Egyptian arithmetic, algebra, and geometry. However, three recent discoveries have changed the picture entirely.

The first of these discoveries was made by A. Seidenberg. He studied the altar constructions in the Indian Śulvasūtras and found that in these relatively ancient texts the "Theorem of Pythagoras" was used to construct a square equal in area to a given rectangle, and that this construction is just that of Euclid. From this and other facts he concluded that Babylonian algebra and geometry and Greek "geometrical algebra" and Hindu geometry are all derived from a common origin, in which altar constructions and the "Theorem of Pythagoras" played a central rôle.

Secondly I have compared the ancient Chinese collection "Nine Chapters of the Arithmetical Art" with Babylonian collections of mathematical problems and found so many similarities that the conclusion of a common pre-Babylonian source seemed unavoidable. In this source, the "Theorem of Pythagoras" must have played a central rôle as well.

The third discovery was made by A. Thom and A. S. Thom, who found that in the construction of megalithic monuments in Southern England and Scotland "Pythagorean Triangles" have been used, that is, right-angled triangles whose sides are integral multiples of a fundamental unit of length. It is well-known that a list of "Pythagorean Triples" like (3,4,5) is found in an ancient Babylonian text, and the Greek and Hindu and Chinese mathematicians also knew how to find such triples.

Combining these three discoveries, I have ventured a tentative reconstruction of a mathematical science which must have existed in the Neolithic Age, say between 3000 and 2500 B.C., and spread from Central Europe to Great Britain, to the Near East, to India, and to China. By far the best account of this mathematical science is found in Chinese texts. My ideas concerning this ancient science will be explained in Chapters 1 and 2.

The Greeks had some knowledge of this ancient science, but they transformed it completely, creating a deductive science based on definitions, postulates and axioms. Yet several traces of pre-Babylonian geometry and algebra can be discerned in the work of Euclid and Diophantos and in popular Greek mathematics. This will be shown in Chapters 3, 4 and 6.

In the treatises of Hindu astronomers like Āryabhaṭa and Brahmagupta, who lived in the sixth and seventh century A.D., we find methods to solve Diophantine equations such as

$$ax + c = by$$

and

$$x^2 = Dy^2 + 1.$$

These methods are based on the Euclidean algorithm. In Chapter 5 I shall give an account of these methods and discuss their relation to Greek science.

Chapter 7 deals with the work of the excellent Chinese geometer Liu Hui (third century A. D.) and with some mathematical passages in the work of the great Indian astronomer Āryabhaṭa (sixth century). It seems to me that both were influenced by the work of Greek geometers and astronomers like Archimedes and Apollonios. In particular I shall discuss Liu Hui's measurement of the circle and Āryabhaṭa's trigonometry.