

Dirk van Dalen

Logic and Structure

Fourth Edition



Springer

Contents

0	Introduction	1
1	Propositional Logic	5
1.1	Propositions and Connectives	5
1.2	Semantics	15
1.3	Some Properties of Propositional logic	21
1.4	Natural Deduction	30
1.5	Completeness	40
1.6	The Missing Connectives	49
2	Predicate Logic	57
2.1	Quantifiers	57
2.2	Structures	58
2.3	The Language of a Similarity Type	60
2.4	Semantics	68
2.5	Simple Properties of Predicate Logic	73
2.6	Identity	81
2.7	Examples	83
2.8	Natural Deduction	91
2.9	Adding the Existential Quantifier	96
2.10	Natural Deduction and Identity	99
3	Completeness and Applications	103
3.1	The Completeness Theorem	103
3.2	Compactness and Skolem-Löwenheim	111
3.3	Some Model Theory	118
3.4	Skolem Functions or How to Enrich Your Language	135
4	Second Order Logic	143

X Contents

5	Intuitionistic Logic	153
5.1	Constructive Reasoning	153
5.2	Intuitionistic Propositional and Predicate Logic	156
5.3	Kripke Semantics	162
5.4	Some Model Theory	173
6	Normalisation	187
6.1	Cuts	187
6.2	Normalization for Classical Logic	192
6.3	Normalization for Intuitionistic Logic	198
7	Gödel's theorem	209
7.1	Primitive recursive functions	209
7.2	Partial Recursive Functions	218
7.3	Recursively enumerable sets	230
7.4	Some arithmetic	236
7.5	Representability	242
7.6	Derivability	246
7.7	Incompleteness	251
Bibliography		257
Index		259