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Approximate Distributions of Order Statistics

With Applications to Nonparametric Statistics

With 30 Illustrations

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