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# Holomorphic Functions and Integral Representations in Several Complex Variables

With 7 Illustrations



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