

M. Loève

Probability Theory I

4th Edition



Springer-Verlag

New York Heidelberg Berlin

M. Loève

Departments of Mathematics and Statistics
University of California at Berkeley
Berkeley, California 94720

Editorial Board

P. R. Halmos

Managing Editor
University of California
Department of Mathematics
Santa Barbara, California 93106

F. W. Gehring

University of Michigan
Department of Mathematics
Ann Arbor, Michigan 48104

C. C. Moore

University of California at Berkeley
Department of Mathematics
Berkeley, California 94720

AMS Subject Classifications
28–01, 60A05, 60Bxx, 60E05, 60Fxx

Library of Congress Cataloging in Publication Data

Loève, Michel, 1907–
Probability theory.

(Graduate texts in mathematics; 45)

Bibliography p.

Includes index.

1. Probabilities. I. Title. II. Series.

QA273.L63 1977 519.2 76–28332

All rights reserved.

No part of this book may be translated or reproduced in any
form without written permission from Springer-Verlag.

© 1963 by M. Loève

© 1977 by Springer-Verlag Inc.

Softcover reprint of the hardcover 4th edition 1977

Originally published in the University Series in Higher Mathematics
(D. Van Nostrand Company); edited by M. H. Stone, L. Nirenberg, and
S. S. Chern.

ISBN 978-1-4684-9466-2

ISBN 978-1-4684-9464-8 (eBook)

DOI 10.1007/978-1-4684-9464-8

CONTENTS OF VOLUME I

GRADUATE TEXTS IN MATHEMATICS VOL. 45

SECTION	PAGE
INTRODUCTORY PART: ELEMENTARY PROBABILITY THEORY	
I. INTUITIVE BACKGROUND	3
1. Events	3
2. Random events and trials	5
3. Random variables	6
II. AXIOMS; INDEPENDENCE AND THE BERNOULLI CASE	8
1. Axioms of the finite case	8
2. Simple random variables	9
3. Independence	11
4. Bernoulli case	12
5. Axioms for the countable case	15
6. Elementary random variables	17
7. Need for nonelementary random variables	22
III. DEPENDENCE AND CHAINS	24
1. Conditional probabilities	24
2. Asymptotically Bernoullian case	25
3. Recurrence	26
4. Chain dependence	28
*5. Types of states and asymptotic behavior	30
*6. Motion of the system	36
*7. Stationary chains	39
COMPLEMENTS AND DETAILS	42

PART ONE: NOTIONS OF MEASURE THEORY

CHAPTER I: SETS, SPACES, AND MEASURES

1. SETS, CLASSES, AND FUNCTIONS	55
1.1 Definitions and notations	55
1.2 Differences, unions, and intersections	56
1.3 Sequences and limits	57
1.4 Indicators of sets	59

SECTION	PAGE
1.5 Fields and σ -fields	59
1.6 Monotone classes	60
*1.7 Product sets	61
*1.8 Functions and inverse functions	62
*1.9 Measurable spaces and functions	64
*2. TOPOLOGICAL SPACES	65
*2.1 Topologies and limits	66
*2.2 Limit points and compact spaces	69
*2.3 Countability and metric spaces	72
*2.4 Linearity and normed spaces	78
3. ADDITIVE SET FUNCTIONS	83
3.1 Additivity and continuity	83
3.2 Decomposition of additive set functions	87
*4. CONSTRUCTION OF MEASURES ON σ -FIELDS	88
*4.1 Extension of measures	88
*4.2 Product probabilities	91
*4.3 Consistent probabilities on Borel fields	93
*4.4 Lebesgue-Stieltjes measures and distribution functions	96
COMPLEMENTS AND DETAILS	100
CHAPTER II: MEASURABLE FUNCTIONS AND INTEGRATION	
5. MEASURABLE FUNCTIONS	103
5.1 Numbers	103
5.2 Numerical functions	105
5.3 Measurable functions	107
6. MEASURE AND CONVERGENCES	111
6.1 Definitions and general properties	111
6.2 Convergence almost everywhere	114
6.3 Convergence in measure	116
7. INTEGRATION	118
7.1 Integrals	119
7.2 Convergence theorems	125
8. INDEFINITE INTEGRALS; ITERATED INTEGRALS	130
8.1 Indefinite integrals and Lebesgue decomposition	130
8.2 Product measures and iterated integrals	135
*8.3 Iterated integrals and infinite product spaces	137
COMPLEMENTS AND DETAILS	139

SECTION	PAGE
PART TWO: GENERAL CONCEPTS AND TOOLS OF PROBABILITY THEORY	
CHAPTER III: PROBABILITY CONCEPTS	
9. PROBABILITY SPACES AND RANDOM VARIABLES	151
9.1 Probability terminology	151
*9.2 Random vectors, sequences, and functions	155
9.3 Moments, inequalities, and convergences	156
*9.4 Spaces L_r	162
10. PROBABILITY DISTRIBUTIONS	168
10.1 Distributions and distribution functions	168
10.2 The essential feature of pr. theory	172
COMPLEMENTS AND DETAILS	174
CHAPTER IV: DISTRIBUTION FUNCTIONS AND CHARACTERISTIC FUNCTIONS	
11. DISTRIBUTION FUNCTIONS	177
11.1 Decomposition	177
11.2 Convergence of d.f.'s	180
11.3 Convergence of sequences of integrals	182
*11.4 Further extension and convergence of moments	184
*11.5 Discussion	187
*12. CONVERGENCE OF PROBABILITIES ON METRIC SPACES	189
*12.1 Convergence	190
*12.2 Regularity and tightness	193
*12.3 Tightness and relative compactness	195
13. CHARACTERISTIC FUNCTIONS AND DISTRIBUTION FUNCTIONS	198
13.1 Uniqueness	199
13.2 Convergences	202
13.3 Composition of d.f.'s and multiplication of ch.f.'s	206
13.4 Elementary properties of ch.f.'s and first applications	207
14. PROBABILITY LAWS AND TYPES OF LAWS	214
14.1 Laws and types; the degenerate type	214
14.2 Convergence of types	216
14.3 Extensions	218
15. NONNEGATIVE-DEFINITENESS; REGULARITY	218
15.1 Ch.f.'s and nonnegative-definiteness	218
*15.2 Regularity and extension of ch.f.'s	223

SECTION	PAGE
*15.3 Composition and decomposition of regular ch.f.'s	226
COMPLEMENTS AND DETAILS	227

PART THREE: INDEPENDENCE

CHAPTER V: SUMS OF INDEPENDENT RANDOM VARIABLES

16. CONCEPT OF INDEPENDENCE	235
16.1 Independent classes and independent functions	235
16.2 Multiplication properties	238
16.3 Sequences of independent r.v.'s	240
*16.4 Independent r.v.'s and product spaces	242
17. CONVERGENCE AND STABILITY OF SUMS; CENTERING AT EXPECTATIONS AND TRUNCATION	243
17.1 Centering at expectations and truncation	244
17.2 Bounds in terms of variances	246
17.3 Convergence and stability	248
*17.4 Generalization	252
*18. CONVERGENCE AND STABILITY OF SUMS; CENTERING AT MEDIANs AND SYMMETRIZATION	255
*18.1 Centering at medians and symmetrization	256
*18.2 Convergence and stability	260
*19. EXPONENTIAL BOUNDS AND NORMED SUMS	266
*19.1 Exponential bounds	266
*19.2 Stability	270
*19.3 Law of the iterated logarithm	272
COMPLEMENTS AND DETAILS	275

CHAPTER VI: CENTRAL LIMIT PROBLEM

20. DEGENERATE, NORMAL, AND POISSON TYPES	280
20.1 First limit theorems and limit laws	280
*20.2 Composition and decomposition	283
21. EVOLUTION OF THE PROBLEM	286
21.1 The problem and preliminary solutions	286
21.2 Solution of the Classical Limit Problem	290
*21.3 Normal approximation	294

SECTION	PAGE
22. CENTRAL LIMIT PROBLEM; THE CASE OF BOUNDED VARIANCES	300
22.1 Evolution of the problem	300
22.2 The case of bounded variances	302
*23. SOLUTION OF THE CENTRAL LIMIT PROBLEM	308
*23.1 A family of limit laws; the infinitely decomposable laws	308
*23.2 The uan condition	314
*23.3 Central Limit Theorem	319
*23.4 Central convergence criterion	323
23.5 Normal, Poisson, and degenerate convergence	327
*24. NORMED SUMS	331
*24.1 The problem	331
*24.2 Norming sequences	332
*24.3 Characterization of \mathfrak{R}	334
*24.4 Identically distributed summands and stable laws	338
24.5 Lévy representation	343
COMPLEMENTS AND DETAILS	349
CHAPTER VII: INDEPENDENT IDENTICALLY DISTRIBUTED SUMMANDS	
25. REGULAR VARIATION AND DOMAINS OF ATTRACTION	353
25.1 Regular variation	354
25.2 Domains of attraction	360
26. RANDOM WALK	368
26.1 Set-up and basic implications	369
26.2 Dichotomy: recurrence and transience	381
26.3 Fluctuations; exponential identities	388
26.4 Fluctuations; asymptotic behaviour	396
COMPLEMENTS AND DETAILS	401
BIBLIOGRAPHY	407
INDEX	413