

STUDENT MATHEMATICAL LIBRARY
Volume 4

Problems in Mathematical Analysis I

Real Numbers,
Sequences and Series

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