

Fernando Q. Gouvêa

p -adic Numbers

An Introduction

With 15 Figures

Springer-Verlag
Berlin Heidelberg GmbH

Fernando Q. Gouvêa
Department of Mathematics
and Computer Science
Colby College
Waterville, ME 04901, USA

Mathematics Subject Classification (1991): 11-01, 11S-xx

ISBN 978-3-540-56844-5

Library of Congress Cataloging-in-Publication Data
Gouvêa, Fernando, 1957–
p-adic numbers: an introduction/Fernando Gouvêa.
p. cm. – (Universitext)
ISBN 978-3-540-56844-5 ISBN 978-3-662-22278-2 (eBook)
DOI 10.1007/978-3-662-22278-2
1. *p*-adic numbers. I. Title. II. Series.
QA241.G64 1993 512'.74–dc20 93-25593 CIP

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1993
Originally published by Springer-Verlag in 1993

41/3140 – 5 4 3 2 1 0 – Printed on acid-free paper

Contents

Introduction	1
1 Apéritif	5
1.1 Hensel's Analogy	5
1.2 Solving Congruences Modulo p^n	12
1.3 Other Examples	17
2 Foundations	21
2.1 Absolute Values on a Field	21
2.2 Basic Properties	27
2.3 Topology	29
2.4 Algebra	37
3 p-adic Numbers	41
3.1 Absolute Values on $\mathbb{Q}$	41
3.2 Completions	47
3.3 Exploring $\mathbb{Q}_p$	58
3.4 Hensel's Lemma	67
3.5 Local and Global	75
4 Elementary Analysis in $\mathbb{Q}_p$	85
4.1 Sequences and Series	86
4.2 Power Series	88
4.3 Some Elementary Functions	97
4.4 Interpolation	109
5 Vector Spaces and Field Extensions	117
5.1 Normed Vector Spaces over Complete Valued Fields	118
5.2 Finite-dimensional Normed Vector Spaces	123
5.3 Finite Field Extensions	127
5.4 Properties of Finite Extensions	142
5.5 Analysis	153
5.6 Example: Adjoining a p -th Root of Unity	155
5.7 On to $\mathbb{C}_p$	161

6	Analysis in $\mathbb{C}_p$	171
6.1	Almost Everything Extends	171
6.2	Derivatives	175
6.3	Deeper Results on Polynomials and Power Series	177
6.4	Entire Functions	195
6.5	Newton Polygons	199
6.6	Problems	217
A	Hints and Comments on the Problems	221
B	A Brief Glance at the Literature	273
B.1	Texts	273
B.2	Software	274
B.3	Other books	275
	Bibliography	277
	Index	279