

Contents

Preface	v
1 Preliminaries	1
1.1 Vector space	1
1.2 Topological spaces	2
1.3 Metric, metric space	6
1.4 Norm, normed linear space	6
1.5 Modular spaces	7
1.6 Inner product, inner product space	10
1.7 Convergence, Cauchy sequences	11
1.8 Density, separability	12
1.9 Completeness	12
1.10 Subspaces	13
1.11 Products of spaces	14
1.12 Schauder bases	14
1.13 Compactness	15
1.14 Operators (mappings)	16
1.15 Isomorphism, embeddings	18
1.16 Continuous linear functionals	19
1.17 Dual space, weak convergence	20
1.18 The principle of uniform boundedness	21
1.19 Reflexivity	21
1.20 Measure spaces: general extension theory	22
1.21 The Lebesgue measure and integral	29
1.22 Modes of convergence	34
1.23 Systems of seminorms, Hahn–Saks theorem	36

2	Spaces of smooth functions	38
2.1	Multiindices and derivatives	38
2.2	Classes of continuous and smooth functions	39
2.3	Completeness	43
2.4	Separability, bases	45
2.5	Compactness	51
2.6	Continuous linear functionals	55
2.7	Extension of functions	59
3	Lebesgue spaces	62
3.1	$\mathcal{L}^p$ -classes	62
3.2	Lebesgue spaces	66
3.3	Mean continuity	67
3.4	Mollifiers	69
3.5	Density of smooth functions	71
3.6	Separability	71
3.7	Completeness	72
3.8	The dual space	74
3.9	Reflexivity	78
3.10	The space L^∞	78
3.11	Hardy inequalities	83
3.12	Sequence spaces	92
3.13	Modes of convergence	93
3.14	Compact subsets	94
3.15	Weak convergence	95
3.16	Isomorphism of $L^p(\Omega)$ and $L^p(0, \mu(\Omega))$	96
3.17	Schauder bases	97
3.18	Weak Lebesgue spaces	101
3.19	Remarks	104
4	Orlicz spaces	108
4.1	Introduction	108
4.2	Young function, Jensen inequality	109
4.3	Complementary functions	115

4.4	The Δ_2 -condition	119
4.5	Comparison of Orlicz classes	122
4.6	Orlicz spaces	126
4.7	Hölder inequality in Orlicz spaces	131
4.8	The Luxemburg norm	134
4.9	Completeness of Orlicz spaces	137
4.10	Convergence in Orlicz spaces	138
4.11	Separability	143
4.12	The space $E^\Phi(\Omega)$	145
4.13	Continuous linear functionals	151
4.14	Compact subsets of Orlicz spaces	155
4.15	Further properties of Orlicz spaces	161
4.16	Isomorphism properties, Schauder bases	163
4.17	Comparison of Orlicz spaces	166
5	Morrey and Campanato spaces	173
5.1	Introduction	173
5.2	Marcinkiewicz spaces	173
5.3	Morrey and Campanato spaces	176
5.4	Completeness	178
5.5	Relations to Lebesgue spaces	178
5.6	Some lemmas	181
5.7	Embeddings	185
5.8	The John–Nirenberg space	187
5.9	Another definition of the space $JN(Q)$	194
5.10	Spaces $N_{p,\lambda}(Q)$	197
5.11	Miscellaneous remarks	199
6	Banach function spaces	203
6.1	Banach function spaces	203
6.2	Associate space	209
6.3	Absolute continuity of the norm	216
6.4	Reflexivity of Banach function spaces	223
6.5	Separability in Banach function spaces	228

7	Rearrangement-invariant spaces	237
7.1	Nonincreasing rearrangements	237
7.2	Hardy–Littlewood inequality	241
7.3	Resonant measure spaces	243
7.4	Maximal nonincreasing rearrangement	249
7.5	Hardy lemma	251
7.6	Rearrangement-invariant spaces	253
7.7	Hardy–Littlewood–Pólya principle	255
7.8	Luxemburg representation theorem	256
7.9	Fundamental function	259
7.10	Endpoint spaces	264
7.11	Almost-compact embeddings	275
7.12	Gould space	292
8	Lorentz spaces	301
8.1	Definition and basic properties	301
8.2	Embeddings between Lorentz spaces	305
8.3	The associate space	307
8.4	The fundamental function	309
8.5	Absolute continuity of norm	309
8.6	Remarks on $\ \cdot\ _{1,\infty}$	311
9	Generalized Lorentz–Zygmund spaces	313
9.1	Measure-preserving transformations	313
9.2	Basic properties	314
9.3	Nontriviality	317
9.4	Fundamental function	318
9.5	Embeddings between Generalized Lorentz–Zygmund spaces	320
9.6	The associate space	332
9.7	When Generalized Lorentz–Zygmund space is Banach function space	353
9.8	Generalized Lorentz–Zygmund spaces and Orlicz spaces	356
9.9	Absolute continuity of norm	367
9.10	Lorentz–Zygmund spaces	372
9.11	Lorentz–Karamata spaces	373

10 Classical Lorentz spaces	375
10.1 Definition and basic properties	375
10.2 Functional properties	380
10.3 Embeddings	388
10.3.1 Embeddings of type $\Lambda \hookrightarrow \Lambda$	392
10.3.2 Embeddings of type $\Lambda \hookrightarrow \Gamma$	393
10.3.3 Embeddings of type $\Gamma \hookrightarrow \Lambda$	396
10.3.4 Embeddings of type $\Gamma \hookrightarrow \Gamma$	399
10.3.5 The Halperin level function	401
10.3.6 Embeddings of type $\Gamma^{p,\infty}(v) \hookrightarrow \Lambda^q(w)$	404
10.3.7 The single-weight case $\Gamma^{1,\infty}(v) \hookrightarrow \Lambda^1(v)$	406
10.4 Associate spaces	409
10.5 Lorentz and Orlicz spaces	411
10.6 Spaces measuring oscillation	412
10.7 The missing case	425
10.8 Embeddings	427
10.8.1 Embeddings of type $S \hookrightarrow S$	429
10.8.2 Embeddings of type $\Gamma \hookrightarrow S$ and $S \hookrightarrow \Gamma$	431
10.8.3 Embeddings of type $\Lambda \hookrightarrow S$ and $S \hookrightarrow \Lambda$	434
11 Variable-exponent Lebesgue spaces	437
11.1 Introduction	437
11.2 Basic properties	438
11.3 Embedding relations	445
11.4 Density of smooth functions	447
11.5 Reflexivity and uniform convexity	450
11.6 Radon–Nikodým property	453
11.7 Daugavet property	455
Bibliography	459
Index	472

