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Asymptotic Analysis

Linear Ordinary Differential Equations

Translated from the Russian
by Andrew Rodick

With 26 Figures



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