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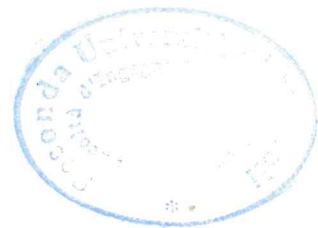
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## Preface to the second edition

The major part of this book is concerned with methods of analysis of stresses and deformations of concrete structures, which are applicable in the design of concrete structures regardless of codes. The second edition includes new sections, an added chapter and more examples. The new examples are worked out in the so-called British units, customary to most engineers in North America. Many other examples in the book are worked out in SI units. The input data and the main results in all examples are given in both SI and British units. It is hoped that by the use of both systems of units the book is equally convenient to readers in all countries. Working out new examples in British units, demonstrating additional applications of the methods presented in the book, is considered more useful than the simpler task of working out each example in both units.

Appendices A and E, dealing with mechanical properties of concrete and with crack width and crack spacing, are based on European and American codes. As expected, the codes have changed after publication of the first edition in 1986. In the second edition the two appendices, almost completely new and greatly expanded, are based on the latest European and American codes. Concrete having strength higher than 50 MPa (7000 psi) and reaching 100 MPa (15 000 psi) is increasingly used in modern structures. The mechanical properties of such a high strength concrete can be predicted by using the graphs and equations of Appendix A.

The new chapter discusses control of cracking in concrete structures. It gives equations to calculate the minimum amount of reinforcement required for crack control.

We would like to take this opportunity to thank readers, especially students at the University of Calgary, who assisted in correcting errors in the first edition, suggested the addition of clarifying material and verified solutions of new examples; we are particularly thankful to Messrs. S. Megally and N. Hammill, graduate students at the University of Calgary. We are also grateful to Mrs. K. Takaoka for the careful typing of the manuscript.

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