use as a reference for approximate solutions, exact solutions of plates and shells are also included. Keeping the current developments in composite-material structures in mind, a brief but reasonably complete, discussion of laminated plates and shells is included in Sections 4.3 and 4.4.

The book contains many example and exercise problems that illustrate, test, and broaden the understanding of the topics covered. A long list of references, by no means complete or up-to-date, is provided in the Bibliography at the end of the book.

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