

Mechanics

Stability of

H. Leipholz

This book is a contribution to the theory of stability of structures, stressing the concepts of stability and instability and the degree of stability. The mathematical treatment has been made in a systematic approach to the problems and the principles of stability are given in a form suitable for the study of such problems. The force systems are treated in a systematic way and have been

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V

This book is a contribution to the understanding of the stressing of structures and instability. It contains concepts and degrees of freedom, mathematical treatment, and has been made available to the reader in a simple and clear manner. The principles of stability are explained in a way that is understandable for those who have been

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Preface

The transition of an elastic system from a stable state into an unstable one is a dynamic process and should be analysed accordingly. There are, however, a number of cases for which this transition consists in the passing through a sequence of nontrivial states of equilibrium. Then, the investigation of the stability problem is possible using statics only. As such cases are numerous in practice, the development of a special theory of stability in elastostatics, initiated by Euler, was called into being. Such development has impeded for a long time the inclusion of 'stability of elastic systems' into a general, uniform theory of stability of dynamic systems. Only in recent decades, specifically under the influence of works by V. V. Bolotin [1] and H. Ziegler [2] it became apparent that stability of elastic systems in the most general sense ought to be regarded from the point of view of dynamics. That is necessary, since in the presence of nonconservative forces the system may not assume nontrivial equilibrium positions at all, and the instability may consist in 'flutter'. Yet flutter, i.e. a vibrational motion with an ever increasing amplitude, can only be described in terms of dynamics.

As a result of this new approach to stability, application of concepts and methods of control theory has become common in elastomechanics. Also, the creation of new mathematical techniques and the adaptation of existing ones was needed. One objective of this book is to investigate such developments and to report on them.

In a number of cases it is not sufficient to evaluate the limit of stability, which may be graspable approximately only. For a satisfactory description of the stability behaviour it may be necessary to investigate also the postcritical behaviour of the system. For that reason, the inclusion of nonlinearities into the calculations is required as well as the observation of load eccentricities and imperfections of the system's geometric shape. Also, this set of problems will be discussed in this book.

A completely new point of view must be adopted if, by virtue of unavoidable inaccuracies of technical data and the lack of a deterministic