



DYNAMICS OF STRUCTURES
International Editions 1993

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CONTENTS

Preface	xv
List of Symbols	xvii
1 Overview of Structural Dynamics	1
1-1 Fundamental Objective of Structural Dynamics Analysis	1
1-2 Types of Prescribed Loadings	2
1-3 Essential Characteristics of a Dynamic Problem	3
1-4 Methods of Discretization	4
Lumped-Mass Procedure	4
Generalized Displacements	5
The Finite-Element Concept	7
1-5 Formulation of the Equations of Motion	9
Direct Equilibration using d'Alembert's Principle	9
Principle of Virtual Displacements	10
Variational Approach	10
1-6 Organization of the Text	11
PART I SINGLE-DEGREE-OF-FREEDOM SYSTEMS	
2 Analysis of Free Vibrations	15
2-1 Components of the Basic Dynamic System	15
2-2 Equation of Motion of the Basic Dynamic System	16
2-3 Influence of Gravitational Forces	17
2-4 Influence of Support Excitation	18
2-5 Analysis of Undamped Free Vibrations	20
2-6 Damped Free Vibrations	25
Critically-Damped Systems	26
Undercritically-Damped Systems	27
Overcritically-Damped Systems	32
Problems	32

3 Response to Harmonic Loading	33
3-1 Undamped System	33
Complementary Solution	33
Particular Solution	33
General Solution	34
3-2 System with Viscous Damping	36
3-3 Resonant Response	42
3-4 Accelerometers and Displacement Meters	45
3-5 Vibration Isolation	46
3-6 Evaluation of Viscous-Damping Ratio	52
Free-Vibration Decay Method	52
Resonant Amplification Method	53
Half-Power (Band-Width) Method	54
Resonance Energy Loss Per Cycle Method	56
3-7 Complex-Stiffness Damping	58
Problems	61
4 Response to Periodic Loading	65
4-1 Fourier Series Expressions of Periodic Loading	65
Trigonometric Form	65
Exponential Form	66
4-2 Response to the Fourier Series Loading	67
4-3 Preview of Frequency-Domain Analysis	69
Problems	71
5 Response to Impulsive Loading	73
5-1 General Nature of Impulsive Loading	73
5-2 Sine-Wave Impulse	74
5-3 Rectangular Impulse	77
5-4 Triangular Impulse	78
5-5 Shock or Response Spectra	79
5-6 Approximate Analysis of Impulsive-Load Response	82
Problems	84
6 Response to General Dynamic Loading: Superposition Methods	87
6-1 Analysis Through the Time Domain	87
Formulation of Response Integral	87
Numerical Evaluation of Response Integral	89
6-2 Analysis Through the Frequency Domain	97
Fourier Response Integral	98
Discrete Fourier Transforms (DFT)	100

Fast Fourier Transforms (FFTs)	102
Evaluation of Dynamic Response	106
6-3 Relationship between the Time- and Frequency-Domain Transfer Functions	109
Problems	109
7 Response to General Dynamic Loading: Step-by-Step Methods	111
7-1 General Concepts	111
7-2 Piecewise Exact Method	112
7-3 Numerical Approximation Procedures – General Comments	116
7-4 Second Central Difference Formulation	117
7-5 Integration Methods	120
Euler-Gauss Procedure	120
Newmark Beta Methods	121
Conversion to Explicit Formulation	123
7-6 Incremental Formulation for Nonlinear Analysis	124
7-7 Summary of the Linear Acceleration Procedure	127
Problems	132
8 Generalized Single-Degree-of-Freedom Systems	133
8-1 General Comments on SDOF Systems	133
8-2 Generalized Properties: Assemblages of Rigid Bodies	134
8-3 Generalized Properties: Distributed Flexibility	140
8-4 Expressions for Generalized System Properties	145
8-5 Vibration Analysis by Rayleigh's Method	149
8-6 Selection of the Rayleigh Vibration Shape	152
8-7 Improved Rayleigh Method	156
Problems	160
PART II MULTI-DEGREE-OF-FREEDOM SYSTEMS	
9 Formulation of the MDOF Equations of Motion	169
9-1 Selection of the Degrees of Freedom	169
9-2 Dynamic-Equilibrium Condition	171
9-3 Axial-Force Effects	173
10 Evaluation of Structural-Property Matrices	175
10-1 Elastic Properties	175
Flexibility	175
Stiffness	176
Basic Structural Concepts	177
Finite-Element Stiffness	179

10-2	Mass Properties	184
	Lumped-Mass Matrix	184
	Consistent-Mass Matrix	185
10-3	Damping Properties	189
10-4	External Loading	189
	Static Resultants	190
	Consistent Nodal Loads	190
10-5	Geometric Stiffness	191
	Linear Approximation	191
	Consistent Geometric Stiffness	194
10-6	Choice of Property Formulation	196
	Problems	198
	11 Undamped Free Vibrations	201
11-1	Analysis of Vibration Frequencies	201
11-2	Analysis of Vibration Mode Shapes	204
11-3	Flexibility Formulation of Vibration Analysis	208
11-4	Influence of Axial Forces	208
	Free Vibrations	208
	Buckling Load	209
	Buckling with Harmonic Excitation	210
11-5	Orthogonality Conditions	211
	Basic Conditions	211
	Additional Relationships	212
	Normalizing	214
	Problems	215
	12 Analysis of Dynamic Response – Using Superposition	219
12-1	Normal Coordinates	219
12-2	Uncoupled Equations of Motion: Undamped	221
12-3	Uncoupled Equations of Motion: Viscous Damping	222
12-4	Response Analysis by Mode Displacement Superposition	223
	Viscous Damping	223
	Complex-Stiffness Damping	230
12-5	Construction of Proportional Viscous Damping Matrices	234
	Rayleigh Damping	234
	Extended Rayleigh Damping	237
	Alternative Formulation	240
	Construction of Nonproportional Damping Matrices	242
12-6	Response Analysis using Coupled Equations of Motion	245
	Time Domain	245

	Frequency Domain	246
12-7	Relationship between Time- and Frequency- Domain Transfer Functions	247
12-8	Practical Procedure for solving Coupled Equations of Motion	251
12-9	Interpolation Procedure for Generation of Transfer Functions	254
	Problems	256
	13 Vibration Analysis by Matrix Iteration	259
13-1	Preliminary Comments	259
13-2	Fundamental Mode Analysis	260
13-3	Proof of Convergence	265
13-4	Analysis of Higher Modes	267
	Second-Mode Analysis	267
	Analysis of Third and Higher Modes	271
	Analysis of Highest Mode	272
13-5	Buckling Analysis by Matrix Iteration	275
13-6	Inverse Iteration – the Preferred Procedure	279
13-7	Inverse Iteration with Shifts	281
13-8	Special Eigenproblem Topics	285
	Eigenproperty Expansion	286
	Symmetric Form of Dynamic Matrix	288
	Analysis of Unconstrained Structures	290
	Problems	291
	14 Selection of Dynamic Degrees of Freedom	293
14-1	Finite-Element Degrees of Freedom	293
	One-Dimensional Elements	294
	Two- and Three-Dimensional Elements	294
14-2	Kinematic Constraints	295
14-3	Static Condensation	296
14-4	Rayleigh Method in Discrete Coordinates	298
14-5	Rayleigh-Ritz Method	299
14-6	Subspace Iteration	304
14-7	Reduction of Modal Truncation Errors	306
	General Comments on Coordinate Reduction	306
	Modal Contributions	307
	Static Correction Procedure	311
	Mode Acceleration Method	313
14-8	Derived Ritz Vectors	314
	Preliminary Comments	314
	Derivation Details	316
	Tridiagonal Equations of Motion	319

Loss of Orthogonality	322
Required Number of Vectors	323
Problems	323
15 Analysis of MDOF Dynamic Response: Step-by-Step Methods	325
15-1 Preliminary Comments	325
15-2 Incremental Equations of Motion	327
15-3 Step-by-Step Integration: Constant Average Acceleration Method	328
15-4 Step-by-Step Integration: Linear Acceleration Method	330
15-5 Strategies for Analysis of Coupled MDOF Systems	332
Localized Nonlinearity	332
Coupled Effects Treated as Pseudo-Forces	336
16 Variational Formulation of the Equations of Motion	341
16-1 Generalized Coordinates	341
16-2 Hamilton's Principle	342
16-3 Lagrange's Equations of Motion	344
16-4 Derivation of the General Equations of Motion for Linear Systems	351
16-5 Constraints and Lagrange Multipliers	356
Problems	359
PART III DISTRIBUTED-PARAMETER SYSTEMS	
17 Partial Differential Equations of Motion	365
17-1 Introduction	365
17-2 Beam Flexure: Elementary Case	366
17-3 Beam Flexure: Including Axial-Force Effects	368
17-4 Beam Flexure: Including Viscous Damping	369
17-5 Beam Flexure: Generalized Support Excitations	370
17-6 Axial Deformations: Undamped	373
Problems	375
18 Analysis of Undamped Free Vibrations	377
18-1 Beam Flexure: Elementary Case	377
18-2 Beam Flexure: Including Axial-Force Effects	386
18-3 Beam Flexure: With Distributed Elastic Support	388
18-4 Beam Flexure: Orthogonality of Vibration Mode Shapes	389
18-5 Free Vibrations in Axial Deformation	391
18-6 Orthogonality of Axial Vibration Modes	392
Problems	394

19 Analysis of Dynamic Response	397
19-1 Normal Coordinates	397
19-2 Uncoupled Flexural Equations of Motion: Undamped Case	400
19-3 Uncoupled Flexural Equations of Motion: Damped Case	403
19-4 Uncoupled Axial Equations of Motion: Undamped Case	407
19-5 Wave-Propagation Analysis	411
Basic Axial-Wave-Propagation Equation	411
Consideration of Boundary Conditions	415
Discontinuity in Bar Properties	418
Problems	423
PART IV RANDOM VIBRATIONS	
20 Probability Theory	427
20-1 Single Random Variable	427
20-2 Important Averages of a Single Random Variable	432
20-3 One-Dimensional Random Walk	434
20-4 Two Random Variables	442
20-5 Important Averages of Two Random Variables	451
20-6 Scatter Diagram and Correlation of Two Random Variables	455
20-7 Principal Axes of Joint Probability Density Function	458
20-8 Rayleigh Probability Density Function	461
20-9 m Random Variables	463
20-10 Linear Transformations of Normally Distributed Random Variables	465
Problems	466
21 Random Processes	471
21-1 Definition	471
21-2 Stationary and Ergodic Processes	473
21-3 Autocorrelation Function for Stationary Processes	478
21-4 Power Spectral Density Function for Stationary Processes	484
21-5 Relationship Between Power Spectral Density and Autocorrelation Functions	485
21-6 Power Spectral Density and Autocorrelation Functions for Derivatives of Processes	488
21-7 Superposition of Stationary Processes	490
21-8 Stationary Gaussian Processes: One Independent Variable	492
21-9 Stationary White Noise	498
21-10 Probability Distribution for Maxima	501
21-11 Probability Distribution for Extreme Values	506

21-12 Nonstationary Gaussian Processes	510
21-13 Stationary Gaussian Process: Two or More Independent Variables	511
Problems	512
22 Stochastic Response of Linear SDOF Systems	517
22-1 Transfer Functions	517
22-2 Relationship between Input and Output Autocorrelation Functions	518
22-3 Relationship between Input and Output Power Spectral Density Functions	522
22-4 Response Characteristics for Narrowband Systems	524
22-5 Nonstationary Mean Square Response Resulting from Zero Initial Conditions	528
22-6 Fatigue Predictions for Narrowband Systems	532
Problems	535
23 Stochastic Response of Linear MDOF Systems	539
23-1 Time-Domain Response for Linear Systems using Normal Modes	539
23-2 Frequency-Domain Response for Linear Systems using Normal Modes	541
23-3 Normal Mode Forcing Function due to Discrete Loadings	543
23-4 Normal Mode Forcing Function due to Distributed Loadings	547
23-5 Frequency-Domain Response for Linear Systems having Frequency-Dependent Parameters and/or Coupled Normal Modes	548
Problems	550

PART V EARTHQUAKE ENGINEERING

24 Seismological Background	555
24-1 Introductory Note	555
24-2 Seismicity	556
24-3 Earthquake Faults and Waves	558
24-4 Structure of the Earth	559
24-5 Plate Tectonics	563
24-6 Elastic-Rebound Theory of Earthquakes	567
24-7 Measures of Earthquake Size	571
25 Free-Field Surface Ground Motions	575
25-1 Fourier and Response Spectra	575
25-2 Factors influencing Response Spectra	581
25-3 Design Response Spectra	586
Dual Strategy of Seismic Design	586
Peak Ground Accelerations	587
Response Spectrum Shapes	590

Uniform-Hazard Site-Specific Response Spectra	595
Two Horizontal Components of Motion	597
25-4 Design Accelerograms	597
Response Spectrum Compatible Accelerograms	598
Principal Axes of Motion	603
Spatially Correlated Motions	607
26 Deterministic Earthquake Response: Systems on Rigid Foundations	613
26-1 Types of Earthquake Excitation	613
26-2 Response to Rigid-Soil Excitations	615
Lumped SDOF Elastic Systems, Translational Excitation	615
Generalized-Coordinate SDOF Elastic Systems, Translational Excitation	617
Lumped MDOF Elastic Systems, Translational Excitation	623
Comparison with ATC-3 Recommended Code Provisions	638
Distributed-Parameter Elastic Systems, Translational Excitation	640
Lumped MDOF Elastic Systems, Rotational Excitation	642
Lumped MDOF Elastic Systems, Multiple Excitation	644
Lumped SDOF Elastic-Plastic Systems, Translational Excitation	647
26-3 Combining Maximum Modal Responses	650
Mean Square Response of a Single Mode	650
Covariance of Response Produced by Two Modes	652
SRSS and CQC Combination of Modal Responses	653
Combining Two-Component-Excitation Responses	657
Problems	662
27 Deterministic Earthquake Response: Including Soil-Structure Interaction	669
27-1 Soil-Structure Interaction by Direct Analysis	669
Kinematic Interaction for Translational Excitation; the Tau Effect	670
Direct Inclusion of a Bounded Soil Layer	673
27-2 Substructure Analysis of SSI Response	674
Lumped SDOF Systems on Rigid Mat Foundation	674
General MDOF System with Multiple Support Excitation	679
Generation of Boundary Impedances	689
27-3 Response of Underground Structures	704
Free-Field Ground Motions due to Propagating Plane Waves	704
Racking Deformations of Cross Sections	705
Overall Axial and Flexural Deformations	706
Influence of Transverse Joints on Axial Deformations	709

28 Stochastic Structural Response
 28-1 Modeling of Strong Ground Motions
 28-2 Stochastic Response of Linear Systems
 SDOF Systems
 MDOF Systems
 28-3 Extreme-Value Response of Nonlinear Systems
 SDOF Systems
 MDOF Systems
 28-4 Design Considerations
 28-5 Allowable Ductility Demand Versus Ductility Capacity

Index

711
 711
 711
 711
 712
 713
 713
 723
 726
 729
 731

PREFACE

Since the first edition of this book was published in 1975, major advances have been made in the subject "Dynamics Of Structures." While it would be impossible to give a comprehensive treatment of all such changes in this second edition, those considered to be of most practical significance are included.

The general organization of text material remains unchanged from the first edition. It progresses logically from a treatment of single-degree-of-freedom systems to multi-degree-of-freedom discrete-parameter systems and then on to infinite-degree-of-freedom continuous systems. The concept of force equilibrium, which forms the basis of static analysis of structures, is retained so that the experienced engineer can easily make the transition to performing a dynamic analysis. It is essential therefore that the student of structural dynamics have a solid background in the theories of statics of structures, including matrix methods, and it is assumed that the readers of this text have such preparation.

The theoretical treatment in Parts I, II, and III is deterministic in nature because it makes use of dynamic loadings which are fully prescribed even though they may be highly irregular and transient with respect to time. The treatment of random vibrations in Part IV is however stochastic (random) in form since the loadings considered can be characterized only in a statistical manner. An understanding of basic probability theory is therefore an essential prerequisite to the study of this subject. Before proceeding with this study, it is recommended that the student take a full course on probability theory; however, if this has not been done, the brief treatment of probability concepts given in Chapter 20 can serve as minimum preparation.

The solution of a typical structural dynamics problem is considerably more complicated than its static counterpart due to the addition of inertia and damping to the elastic resistance forces and due to the time dependency of all force quantities. For most practical situations, the solution usually is possible only through the use of a high-speed digital computer, which has become the standard tool of the structural dynamicist. However, most of the problems in the text, which are intended to teach the fundamentals of dynamics, are quite simple in form allowing their solutions to be obtained using a hand calculator. Nevertheless, the student of dynamics of structures should have previously studied computer coding techniques and the associated analytical procedures. Such background will permit an early transition from solving dynamics problems by hand calculator to solving them on a PC computer using programs specially developed for this purpose. The program CAL-91, developed by Professor E. L. Wilson of the University of California, Berkeley, is such a program which has been used very effectively in teaching even the first course in Dynamics Of Structures. Instructors using this book are encour-