Chapter 1 gives the introduction to buckling and the importance of the critical buckling load in design. Chapter 2 presents the flexural buckling solutions for columns under various loading, restraints and boundary conditions. The effect of transverse shear deformation on the buckling load of columns, a brief discussion on the flexural–torsional buckling of columns for thin-walled members with open-profile, and inelastic buckling of columns are presented in Chapter 2. Chapter 3 contains the exact flexural–torsional buckling solutions of beams and the buckling solutions of circular arches and rings. Chapter 4 deals with the buckling of thin and thick plates under inplane loads for various shapes and boundary conditions. Results for inelastic buckling of circular, rectangular and polygonal plates are also presented. Finally, Chapter 5 presents buckling solutions for cylindrical shells, spherical shells and truncated conical shells.

It is hoped that this book will be a useful reference source for benchmark solutions that is so needed in checking the validity, accuracy and convergence of numerical results.

This book contains so many mathematical equations and numbers that it is impossible not to have typographical and other kinds of errors. We wish to thank in advance those readers who are willing to draw attention to typos and errors, using the e-mail addresses: *cvewcm@nus.edu.sg*, *cywang@math.msu.edu*, or *jnreddy@tamu.edu*.

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