Contents

PREFACE ***xix***

ACKNOWLEDGMENTS ***xxi***

1 INTRODUCTION ***1***

1.1 Digital Image Processing: Problems and Applications 1
1.2 Image Representation and Modeling 4
1.3 Image Enhancement 6
1.4 Image Restoration 7
1.5 Image Analysis 7
1.6 Image Reconstruction from Projections 8
1.7 Image Data Compression 9
Bibliography 10

2 TWO-DIMENSIONAL SYSTEMS AND MATHEMATICAL PRELIMINARIES ***11***

2.1 Introduction 11
2.2 Notation and Definitions 11
2.3 Linear Systems and Shift Invariance 13
2.4 The Fourier Transform 15
Properties of the Fourier Transform, 16
Fourier Transform of Sequences (Fourier Series), 18

2.5 The Z-Transform or Laurent Series 20
Causality and Stability, 21
2.6 Optical and Modulation Transfer Functions 21
2.7 Matrix Theory Results 22
Vectors and Matrices, 22
Row and Column Ordering, 23
Transposition and Conjugation Rules, 25
Toeplitz and Circulant Matrices, 25
Orthogonal and Unitary Matrices, 26
Positive Definiteness and Quadratic Forms, 27
Diagonal Forms, 27
2.8 Block Matrices and Kronecker Products 28
Block Matrices, 28
Kronecker Products, 30
Separable Operations, 31
2.9 Random Signals 31
Definitions, 31
Gaussian or Normal Distribution, 32
Gaussian Random Processes, 32
Stationary Processes, 32
Markov Processes, 33
Orthogonality and Independence, 34
The Karhunen Loève (KL) Transform, 34
2.10 Discrete Random Fields 35
Definitions, 35
Separable and Isotropic Covariance Functions, 36
2.11 The Spectral Density Function 37
Properties of the SDF, 38
2.12 Some Results from Estimation Theory 39
Mean Square Estimates, 40
The Orthogonality Principle, 40
2.13 Some Results from Information Theory 41
Information, 42
Entropy, 42
The Rate Distortion Function, 43
Problems 44
Bibliography 47

3 IMAGE PERCEPTION **49**

3.1 Introduction 49
3.2 Light, Luminance, Brightness, and Contrast 49
Simultaneous Contrast, 51
Mach Bands, 53

3.3 MTF of the Visual System 54
3.4 The Visibility Function 55
3.5 Monochrome Vision Models 56
3.6 Image Fidelity Criteria 57
3.7 Color Representation 60
3.8 Color Matching and Reproduction 62
Laws of Color Matching, 63
Chromaticity Diagram , 65
3.9 Color Coordinate Systems 66
3.10 Color Difference Measures 71
3.11 Color Vision Model 73
3.12 Temporal Properties of Vision 75
Bloch's Law, 75
Critical Fusion Frequency (CFF), 75
Spatial versus Temporal Effects, 75
Problems 76
Bibliography 78

4 IMAGE SAMPLING AND QUANTIZATION **80**

4.1 Introduction 80
Image Scanning, 80
Television Standards, 81
Image Display and Recording, 83
4.2 Two-Dimensional Sampling Theory 84
Bandlimited Images, 84
Sampling Versus Replication, 85
Reconstruction of the Image from Its Samples, 85
Nyquist Rate, Aliasing, and Foldover Frequencies, 87
Sampling Theorem, 88
Remarks, 89
4.3 Extensions of Sampling Theory 89
Sampling Random Fields, 90
Sampling Theorem for Random Fields, 90
Remarks, 90
Nonrectangular Grid Sampling and Interlacing, 91
Hexagonal Sampling, 92
Optimal Sampling, 92

4.4 Practical Limitations in Sampling and Reconstruction 93
Sampling Aperture, 93
Display Aperture/Interpolation Function, 94
Lagrange Interpolation, 98
Moire Effect and Flat Field Response, 99

4.5 Image Quantization 99

4.6 The Optimum Mean Square or Lloyd-Max Quantizer 101
The Uniform Optimal Quantizer, 103
Properties of the Optimum Mean Square Quantizer, 103
Proofs, 112

4.7 A Compandor Design 113
Remarks, 114

4.8 The Optimum Mean Square Uniform Quantizer for Nonuniform Densities 115

4.9 Examples, Comparison, and Practical Limitations 115

4.10 Analytic Models for Practical Quantizers 118

4.11 Quantization of Complex Gaussian Random Variables 119

4.12 Visual Quantization 119
Contrast Quantization, 120
Pseudorandom Noise Quantization, 120
Halftone Image Generation, 121
Color Quantization, 122

Problems 124

Bibliography 128

5 IMAGE TRANSFORMS 132

5.1 Introduction 132

5.2 Two-Dimensional Orthogonal and Unitary Transforms 134
Separable Unitary Transforms, 134
Basis Images, 135
Kronecker Products and Dimensionality, 137
Dimensionality of Image Transforms, 138
Transform Frequency, 138
Optimum Transform, 138

5.3 Properties of Unitary Transforms 138
Energy Conservation and Rotation, 138
Energy Compaction and Variances of Transform Coefficients, 139
Decorrelation, 140
Other Properties, 140

5.4 The One-Dimensional Discrete Fourier Transform (DFT) 141
Properties of the DFT/Unitary DFT, 141

5.5 The Two-Dimensional DFT 145
Properties of the Two-Dimensional DFT, 147

5.6 The Cosine Transform 150
Properties of the Cosine Transform, 151

5.7 The Sine Transform 154
Properties of the Sine Transform, 154

5.8 The Hadamard Transform 155
Properties of the Hadamard Transform, 157

5.9 The Haar Transform 159
Properties of the Haar Transform, 161

5.10 The Slant Transform 161
Properties of the Slant Transform, 162

5.11 The KL Transform 163
KL Transform of Images, 164
Properties of the KL Transform, 165

5.12 A Sinusoidal Family of Unitary Transforms 175
Approximation to the KL Transform, 176

5.13 Outer Product Expansion and Singular Value Decomposition 176
Properties of the SVD Transform, 177

5.14 Summary 180

Problems 180

Bibliography 185

6 IMAGE REPRESENTATION BY STOCHASTIC MODELS 189

6.1 Introduction 189
Covariance Models, 189
Linear System Models, 189

6.2 One-Dimensional Causal Models 190
Autoregressive (AR) Models, 190
Properties of AR Models, 191
Application of AR Models in Image Processing, 193
Moving Average (MA) Representations, 194
Autoregressive Moving Average (ARMA) Representations, 195
State Variable Models, 195
Image Scanning Models, 196

6.3 One-Dimensional Spectral Factorization 196
Rational SDFs, 197
Remarks, 198

6.4 AR Models, Spectral Factorization, and Levinson Algorithm 198
The Levinson-Durbin Algorithm, 198

6.5 Noncausal Representations 200
Remarks, 201
Noncausal MVRs for Autoregressive Sequences, 201
A Fast KL Transform, 202
Optimum Interpolation of Images, 204

6.6 Linear Prediction in Two Dimensions 204
Causal Prediction, 205
Semicausal Prediction, 206
Noncausal Prediction, 206
Minimum Variance Prediction, 206
Stochastic Representation of Random Fields, 207
Finite-Order MVRs, 208
Remarks, 209
Stability of Two-Dimensional Systems, 212

6.7 Two-Dimensional Spectral Factorization and Spectral Estimation Via Prediction Models 213
Separable Models, 214
Realization of Noncausal MVRs, 215
Realization of Causal and Semicausal MVRs, 216
Realization via Orthogonality Condition, 216

6.8 Spectral Factorization via the Wiener-Doob Homomorphic Transformation 219
Causal MVRs, 220
Semicausal WNDRs, 220
Semicausal MVRs, 222
Remarks and Examples, 222

6.9 Image Decomposition, Fast KL Transforms, and Stochastic Decoupling 223
Periodic Random Fields, 223
Noncausal Models and Fast KL Transforms, 224
Semicausal Models and Stochastic Decoupling, 225

6.10 Summary 226

Problems 227

Bibliography 230

7 IMAGE ENHANCEMENT 233

7.1 Introduction 233

7.2 Point Operations 235
Contrast Stretching, 235
Clipping and Thresholding, 235
Digital Negative, 238
Intensity Level Slicing, 238
Bit Extraction, 239
Range Compression, 240
Image Subtraction and Change Detection, 240

7.3 Histogram Modeling 241
Histogram Equalization, 241
Histogram Modification, 242
Histogram Specification, 243

7.4 Spatial Operations 244
Spatial Averaging and Spatial Low-pass Filtering, 244
Directional Smoothing, 245
Median Filtering, 246
Other Smoothing Techniques, 249
Unsharp Masking and Crispening, 249
Spatial Low-pass, High-pass and Band-pass Filtering, 250
Inverse Contrast Ratio Mapping and Statistical Scaling, 252
Magnification and Interpolation (Zooming), 253
Replication, 253
Linear Interpolation, 253

7.5 Transform Operations 256
Generalized Linear Filtering, 256
Root Filtering, 258
Generalized Cepstrum and Homomorphic Filtering, 259

7.6 Multispectral Image Enhancement 260
Intensity Ratios, 260
Log-Ratios, 261
Principal Components, 261

7.7 False Color and Pseudocolor 262

7.8 Color Image Enhancement 262

7.9 Summary 263

Problems 263

Bibliography 265

8 IMAGE FILTERING AND RESTORATION 267

8.1 Introduction 267

8.2 Image Observation Models 268
Image Formation Models, 269
Detector and Recorder Models, 273
Noise Models, 273
Sampled Image Observation Models, 275

8.3 Inverse and Wiener Filtering 275
Inverse Filter, 275
Pseudoinverse Filter, 276
The Wiener Filter, 276
Remarks, 279

8.4 Finite Impulse Response (FIR) Wiener Filters 284
Filter Design, 284
Remarks, 285
Spatially Varying FIR Filters, 287

8.5 Other Fourier Domain Filters 290
Geometric Mean Filter, 291
Nonlinear Filters, 291

8.6 Filtering Using Image Transforms 292
Wiener Filtering, 292
Remarks, 293
Generalized Wiener Filtering, 293
Filtering by Fast Decompositions, 295

8.7 Smoothing Splines and Interpolation 295
Remarks, 297

8.8 Least Squares Filters 297
Constrained Least Squares Restoration, 297
Remarks, 298

8.9 Generalized Inverse, SVD, and Iterative Methods 299
The Pseudoinverse, 299
Minimum Norm Least Squares (MNLS) Solution and the Generalized Inverse, 300
One-step Gradient Methods, 301
Van Cittert Filter, 301
The Conjugate Gradient Method, 302
Separable Point Spread Functions, 303

8.10 Recursive Filtering For State Variable Systems 304
Kalman Filtering, 304
Remarks, 307

8.11 Causal Models and Recursive Filtering 307
A Vector Recursive Filter, 308
Stationary Models, 310
Steady-State Filter, 310
A Two-Stage Recursive Filter, 310
A Reduced Update Filter, 310
Remarks, 311

8.12 Semicausal Models and Semirecursive Filtering 311
Filter Formulation, 312

8.13 Digital Processing of Speckle Images 313
Speckle Representation, 313
Speckle Reduction: N-Look Method, 315
Spatial Averaging of Speckle, 315
Homomorphic Filtering, 315

8.14 Maximum Entropy Restoration 316
Distribution-Entropy Restoration, 317
Log-Entropy Restoration, 318

8.15 Bayesian Methods 319
Remarks, 320

8.16 Coordinate Transformation and Geometric Correction 320

8.17 Blind Deconvolution 322

8.18 Extrapolation of Bandlimited Signals 323
Analytic Continuation, 323
Super-resolution, 323
Extrapolation Via Prolate Spheroidal Wave Functions (PSWFs), 324
Extrapolation by Error Energy Reduction, 324
Extrapolation of Sampled Signals, 326
Minimum Norm Least Squares (MNLS) Extrapolation, 326
Iterative Algorithms, 327
Discrete Prolate Spheroidal Sequences (DPSS), 327
Mean Square Extrapolation, 328
Generalization to Two Dimensions, 328

8.19 Summary 330

Problems 331

Bibliography 335

9 IMAGE ANALYSIS AND COMPUTER VISION 342

9.1 Introduction 342

9.2 Spatial Feature Extraction 344
Amplitude Features, 344
Histogram Features, 344

9.3 Transform Features 346

9.4 Edge Detection 347
Gradient Operators, 348
Compass Operators, 350
Laplace Operators and Zero Crossings, 351
Stochastic Gradients, 353
Performance of Edge Detection Operators, 355
Line and Spot Detection, 356

9.5 Boundary Extraction 357
Connectivity, 357
Contour Following, 358
Edge Linking and Heuristic Graph Searching, 358
Dynamic Programming, 359
Hough Transform, 362

9.6 Boundary Representation 362
Chain Codes, 363
Fitting Line Segments, 364
B-Spline Representation, 364
Fourier Descriptors, 370
Autoregressive Models, 374

9.7 Region Representation 375
Run-length Codes, 375
Quad-Trees, 375
Projections, 376

9.8 Moment Representation 377
Definitions, 377
Moment Representation Theorem, 378
Moment Matching, 378
Orthogonal Moments, 379
Moment Invariants, 380
Applications of Moment Invariants, 381

9.9 Structure 381
Medial Axis Transform, 381
Morphological Processing, 384
Morphological Transforms, 387

9.10 Shape Features 390
Geometry Features, 391
Moment-Based Features, 392

9.11 Texture 394
Statistical Approaches, 394
Structural Approaches, 398
Other Approaches, 399

9.12 Scene Matching and Detection 400
Image Subtraction, 400
Template Matching and Area Correlation, 400
Matched Filtering, 403
Direct Search Methods, 404

9.13 Image Segmentation 407
Amplitude Thresholding or Window Slicing, 407
Component Labeling, 409
Boundary-based Approaches, 411
Region-based Approaches and Clustering, 412
Template Matching, 413
Texture Segmentation, 413

9.14 Classification Techniques 414
Supervised Learning, 414
Nonsupervised Learning or Clustering, 418

9.15 Image Understanding 421

Problems 422

Bibliography 425

10 IMAGE RECONSTRUCTION FROM PROJECTIONS 431

10.1 Introduction 431
Transmission Tomography, 431
Reflection Tomography, 432
Emission Tomography, 433
Magnetic Resonance Imaging, 434
Projection-based Image Processing, 434

10.2 The Radon Transform 434
Definition, 434
Notation, 436
Properties of the Radon Transform, 437

10.3 The Back-projection Operator 439
Definition, 439
Remarks, 440

10.4 The Projection Theorem 442
Remarks, 443

10.5 The Inverse Radon Transform 444
Remarks, 445
Convolution Back-projection Method, 446
Filter Back-projection Method, 446
Two-Dimensional Filtering via the Radon Transform, 447

10.6 Convolution/Filter Back-projection Algorithms: Digital Implementation 448
Sampling Considerations, 448
Choice of Filters, 448
Convolution Back-projection Algorithm, 450
Filter Back-projection Algorithm, 451
Reconstruction Using a Parallel Pipeline Processor, 452

10.7 Radon Transform of Random Fields 452
A Unitary Transform $\tilde{R}$, 452
Radon Transform Properties for Random Fields, 456
Projection Theorem for Random Fields, 457

10.8 Reconstruction from Blurred Noisy Projections 458
Measurement Model, 458
The Optimum Mean Square Filter, 458
Remarks, 458

10.9 Fourier Reconstruction 462
Algorithm, 462
Reconstruction of Magnetic Resonance Images, 463

10.10 Fan-Beam Reconstruction 464

10.11 Algebraic Methods 465
The Reconstruction Problem as a Set of Linear Equations, 465
Algebraic Reconstruction Techniques, 466

10.12 Three-Dimensional Tomography 468
Three-Dimensional Reconstruction Algorithms, 469

10.13 Summary 470

Problems 470

Bibliography 473

11 IMAGE DATA COMPRESSION **476**

11.1 Introduction 476
Image Raw Data Rates, 476
Data Compression versus Bandwidth Compression, 477
Information Rates, 477
Subsampling, Coarse Quantization, Frame Repetition, and Interlacing, 479

11.2 Pixel Coding 479
PCM, 480
Entropy Coding, 480
Run-Length Coding, 481
Bit-Plane Encoding, 483

11.3 Predictive Techniques 483
Basic Principle, 483
Feedback versus Feedforward Prediction, 484
Distortionless Predictive Coding, 485
Performance Analysis of DPCM, 486
Delta Modulation, 488
Line-by-Line DPCM, 490
Two-Dimensional DPCM, 491
Performance Comparisons, 493
Remarks, 494
Adaptive Techniques, 495
Other Methods, 497

11.4 Transform Coding Theory 498
The Optimum Transform Coder, 498
Proofs, 499
Remarks, 501
Bit Allocation and Rate-Distortion Characteristics, 501

11.5 Transform Coding of Images 504
Two-Dimensional Coding Algorithm, 504
Transform Coding Performances Trade-offs and Examples, 507
Zonal versus Threshold Coding, 508
Fast KL Transform Coding, 510
Remarks, 512
Two-Source Coding, 513
Transform Coding under Visual Criterion, 515
Adaptive Transform Coding, 515
Summary of Transform Coding, 516

11.6 Hybrid Coding and Vector DPCM 518
Basic Idea, 518
Adaptive Hybrid Coding, 520
Hybrid Coding Conclusions, 521

11.7 Interframe Coding 521
Frame Repetition, 521
Resolution Exchange, 521
Conditional Replenishment, 522
Adaptive Predictive Coding, 522
Predictive Coding with Motion Compensation, 524
Interframe Hybrid Coding, 527
Three-Dimensional Transform Coding, 529

11.8 Image Coding in the Presence of Channel Errors 532
The Optimum Mean Square Decoder, 532
The Optimum Encoding Rule, 533
Optimization of PCM Transmission, 534
Channel Error Effects in DPCM, 536
Optimization of Transform Coding, 537

11.9 Coding of Two Tone Images 540
Run-length Coding, 540
White Block Skipping, 546
Prediction Differential Quantization, 547
Relative Address Coding, 547
CCITT Modified Relative Element Address Designate Coding, 548
Predictive Coding, 551
Adaptive Predictors, 552
Comparison of Algorithms, 553
Other Methods, 553

11.10 Color and Multispectral Image Coding 553

11.11 Summary 557

Problems 557

Bibliography 561

INDEX ***566***