Berkeley Problems in Mathematics



Contents

Preface

I Problems

| 1 Re | al Analysis | 3 | | | | |
|-----------------------------|---------------------------------|-----------|--|--|--|--|
| 1.1 | Elementary Calculus | 3 | | | | |
| 1.2 | Limits and Continuity | 8 | | | | |
| 1.3 | Sequences, Series, and Products | 10 | | | | |
| 1.4 | Differential Calculus | 13 | | | | |
| 1.5 | Integral Calculus | 17 | | | | |
| 1.6 | Sequences of Functions | 21 | | | | |
| 1.7 | Fourier Series | 26 | | | | |
| 1.8 | Convex Functions | 27 | | | | |
| 2 Multivariable Calculus 29 | | | | | | |
| 2.1 | Limits and Continuity | 29 | | | | |
| 2.2 | Differential Calculus | 30 | | | | |
| 2.3 | Integral Calculus | 37 | | | | |
| 3 Differential Equations 39 | | | | | | |
| 3.1 | First Order Equations | 39 | | | | |
| 3.2 | Second Order Equations | 43 | | | | |

.

| 3.3 | Higher Order Equations |
|-------|--|
| 3.4 | Systems of Differential Equations |
| 4 M | etric Spaces 5 |
| 4.1 | Topology of \mathbb{R}^n |
| 4.2 | General Theory |
| 4.3 | Fixed Point Theorem 5 |
| 5 Co | omplex Analysis 5 |
| 5.1 | Complex Numbers |
| 5.2 | Series and Sequences of Functions |
| 5.3 | Conformal Mappings |
| 5.4 | Integral Representation of Analytic Functions 6 |
| 5.5 | Functions on the Unit Disc |
| 5.6 | Growth Conditions |
| 5.7 | Analytic and Meromorphic Functions |
| 5.8 | Cauchy's Theorem |
| 5.9 | Zeros and Singularities |
| 5.10 | Harmonic Functions |
| 5.11 | Residue Theory |
| 5.12 | Integrals Along the Real Axis |
| | |
| 6 Al | gebra 8 |
| 6.1 | Examples of Groups and General Theory |
| 6.2 | Homomorphisms and Subgroups |
| 6.3 | Cyclic Groups |
| 6.4 | Normality, Quotients, and Homomorphisms 9 |
| 6.5 | S_n, A_n, D_n, \dots 99 |
| 6.6 | Direct Products |
| 6.7 | Free Groups, Products, Generators, and Relations |
| 6.8 | Finite Groups |
| 6.9 | Rings and Their Homomorphisms |
| 6.10 | Ideals |
| 6.11 | Polynomials |
| 6.12 | Fields and Their Extensions |
| 6.13 | Elementary Number Theory |
| 7 T.i | near Algohra 10 |
| 71 | Vector Space 10 |
| 79 | Rank and Determinants |
| 73 | Systems of Fountions |
| 7 / | Linear Transformations |
| 75 | Figenvalues and Figenvectors |
| 76 | Canonical Forms |
| 77 | Similarity |
| 1.1 | JIIIIIaIIiy |

| 7.8 | Bilinear, Quadratic Forms, and Inner Product Spaces | 130 |
|-----|---|-----|
| 7.9 | General Theory of Matrices | 132 |

II Solutions

| 1 Re | eal Analysis | 139 |
|------|---|-----|
| 1.1 | Elementary Calculus | 139 |
| 1.2 | Limits and Continuity | 152 |
| 1.3 | Sequences, Series, and Products | 157 |
| 1.4 | Differential Calculus | 167 |
| 1.5 | Integral Calculus | 174 |
| 1.6 | Sequences of Functions | 184 |
| 1.7 | Fourier Series | 195 |
| 1.8 | Convex Functions | 198 |
| 2 M | ultivariable Calculus | 201 |
| 2.1 | Limits and Continuity | 201 |
| 2.2 | Differential Calculus | 202 |
| 2.3 | Integral Calculus | 215 |
| 2.0 | | |
| 3 Di | fferential Equations | 219 |
| 3.1 | First Order Equations | 219 |
| 3.2 | Second Order Equations | 224 |
| 3.3 | Higher Order Equations | 228 |
| 3.4 | Systems of Differential Equations | 230 |
| | | |
| 4 M | etric Spaces | 237 |
| 4.1 | Topology of \mathbb{R}^n | 237 |
| 4.2 | General Theory | 242 |
| 4.3 | Fixed Point Theorem | 245 |
| 5 Cc | omplex Analysis | 249 |
| 5.1 | Complex Numbers | 249 |
| 5.2 | Series and Sequences of Functions | 252 |
| 5.3 | Conformal Mappings | 256 |
| 5.4 | Integral Representation of Analytic Functions | 260 |
| 5.5 | Functions on the Unit Disc | 262 |
| 5.6 | Growth Conditions | 270 |
| 5.7 | Analytic and Meromorphic Functions | 273 |
| 5.8 | Cauchy's Theorem | 280 |
| 5.9 | Zeros and Singularities | 286 |
| 5.10 | Harmonic Functions | 298 |
| 5.11 | Residue Theory | 299 |
| 5.12 | Integrals Along the Real Axis | 311 |

1

| 6 | Algebra | 335 |
|------------|---|------------|
| 6.1 | 1 Examples of Groups and General Theory | 335 |
| 6.2 | 2 Homomorphisms and Subgroups | 336 |
| 6.3 | \mathbf{B}^{\sim} Cyclic Groups | 338 |
| 6.4 | 4 Normality, Quotients, and Homomorphisms | 339 |
| 6.5 | $5 S_n, A_n, D_n, \dots \dots$ | 341 |
| 6.6 | 5 Direct Products | 343 |
| 6.7 | 7 Free Groups, Products, Generators, and Relations | 345 |
| 6.8 | 8 Finite Groups | 347 |
| 6.9 | 9 Rings and Their Homomorphisms | 350 |
| 6.1 | 10 Ideals | 352 |
| 6.1 | 11 Polynomials | 354 |
| 6.1 | 12 Fields and Their Extensions | 360 |
| 6.1 | 13 Elementary Number Theory | 365 |
| _ | | |
| 7 | Linear Algebra | 371 |
| 7.1 | Vector Spaces | 371 |
| 7.2 | 2 Rank and Determinants | 375 |
| 7.8 | 3 Systems of Equations | 381 |
| 7.4 | 4 Linear Transformations | 382 |
| 7.6 | Eigenvalues and Eigenvectors | 388 |
| 7.0 | | 390 |
| 7.1 | $\begin{array}{cccc} & \text{Similarity} & \dots & $ | 408 |
| 7.8 | 8 Bilinear, Quadratic Forms, and Inner Product Spaces | 411 |
| 7.9 | 9 General Theory of Matrices | 414 |
| | | |
| III | [Appendices | 421 |
| | | |
| A | How to Get the Exams | 423 |
| A . | 1 On-line | 423 |
| А. | 2 Off-line, the Last Resort | 423 |
| в | Passing Scores | 429 |
| ~ | | |
| С | C The Syllabus | |
| References | | 433 |
| Ind | Index | |
| | | |