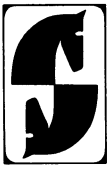
Richard Beals

Advanced Mathematical Analysis

Periodic Functions and Distributions, Complex Analysis, Laplace Transform and Applications



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