



<http://www.springer.com/978-3-540-40386-9>

Mathematical Analysis I

Zorich, V.A.

2004, XVIII, 574 p., Hardcover

ISBN: 978-3-540-40386-9

Table of Contents

1	Some General Mathematical Concepts and Notation	1
1.1	Logical Symbolism	1
1.1.1	Connectives and Brackets	1
1.1.2	Remarks on Proofs	2
1.1.3	Some Special Notation	3
1.1.4	Concluding Remarks	3
1.1.5	Exercises	4
1.2	Sets and Elementary Operations on them	5
1.2.1	The Concept of a Set	5
1.2.2	The Inclusion Relation	7
1.2.3	Elementary Operations on Sets	8
1.2.4	Exercises	10
1.3	Functions	11
1.3.1	The Concept of a Function (Mapping)	11
1.3.2	Elementary Classification of Mappings	15
1.3.3	Composition of Functions. Inverse Mappings	16
1.3.4	Functions as Relations. The Graph of a Function	19
1.3.5	Exercises	22
1.4	Supplementary Material	25
1.4.1	The Cardinality of a Set (Cardinal Numbers)	25
1.4.2	Axioms for Set Theory	27
1.4.3	Set-theoretic Language for Propositions	29
1.4.4	Exercises	31
2	The Real Numbers	35
2.1	Axioms and Properties of Real Numbers	35
2.1.1	Definition of the Set of Real Numbers	35
2.1.2	Some General Algebraic Properties of Real Numbers ..	39
2.1.3	The Completeness Axiom. Least Upper Bound	44
2.2	Classes of Real Numbers and Computations	46
2.2.1	The Natural Numbers. Mathematical Induction	46
2.2.2	Rational and Irrational Numbers	49
2.2.3	The Principle of Archimedes	52
2.2.4	Geometric Interpretation. Computational Aspects	54

XIV Table of Contents

2.2.5	Problems and Exercises	66
2.3	Basic Lemmas on Completeness	70
2.3.1	The Nested Interval Lemma	71
2.3.2	The Finite Covering Lemma	71
2.3.3	The Limit Point Lemma	72
2.3.4	Problems and Exercises	73
2.4	Countable and Uncountable Sets	74
2.4.1	Countable Sets	74
2.4.2	The Cardinality of the Continuum	76
2.4.3	Problems and Exercises	76
3	Limits	79
3.1	The Limit of a Sequence.....	79
3.1.1	Definitions and Examples	79
3.1.2	Properties of the Limit of a Sequence	81
3.1.3	Existence of the Limit of a Sequence	85
3.1.4	Elementary Facts about Series	95
3.1.5	Problems and Exercises	104
3.2	The Limit of a Function	107
3.2.1	Definitions and Examples	107
3.2.2	Properties of the Limit of a Function	111
3.2.3	Limits over a Base.....	127
3.2.4	Existence of the Limit of a Function	131
3.2.5	Problems and Exercises	147
4	Continuous Functions	151
4.1	Basic Definitions and Examples	151
4.1.1	Continuity of a Function at a Point	151
4.1.2	Points of Discontinuity	155
4.2	Properties of Continuous Functions	158
4.2.1	Local Properties	158
4.2.2	Global Properties of Continuous Functions	160
4.2.3	Problems and Exercises	169
5	Differential Calculus	173
5.1	Differentiable Functions	173
5.1.1	Statement of the Problem	173
5.1.2	Functions Differentiable at a Point.....	178
5.1.3	Tangents. Geometric Meaning of the Derivative	181
5.1.4	The Role of the Coordinate System	184
5.1.5	Some Examples	185
5.1.6	Problems and Exercises	191
5.2	The Basic Rules of Differentiation	193
5.2.1	Differentiation and the Arithmetic Operations	193
5.2.2	Differentiation of a Composite Function (chain rule) ..	196

5.2.3	Differentiation of an Inverse Function	199
5.2.4	Table of Derivatives of Elementary Functions	204
5.2.5	Differentiation of a Very Simple Implicit Function	204
5.2.6	Higher-order Derivatives	209
5.2.7	Problems and Exercises	212
5.3	The Basic Theorems of Differential Calculus	214
5.3.1	Fermat's Lemma and Rolle's Theorem	214
5.3.2	The theorems of Lagrange and Cauchy	216
5.3.3	Taylor's Formula	219
5.3.4	Problems and Exercises	232
5.4	Differential Calculus Used to Study Functions	236
5.4.1	Conditions for a Function to be Monotonic	236
5.4.2	Conditions for an Interior Extremum of a Function	237
5.4.3	Conditions for a Function to be Convex	243
5.4.4	L'Hôpital's Rule	250
5.4.5	Constructing the Graph of a Function	252
5.4.6	Problems and Exercises	261
5.5	Complex Numbers and Elementary Functions	265
5.5.1	Complex Numbers	265
5.5.2	Convergence in \mathbb{C} and Series with Complex Terms	268
5.5.3	Euler's Formula and the Elementary Functions	273
5.5.4	Power Series Representation. Analyticity	276
5.5.5	Algebraic Closedness of the Field \mathbb{C}	282
5.5.6	Problems and Exercises	287
5.6	Examples of Differential Calculus in Natural Science	289
5.6.1	Motion of a Body of Variable Mass	289
5.6.2	The Barometric Formula	291
5.6.3	Radioactive Decay and Nuclear Reactors	293
5.6.4	Falling Bodies in the Atmosphere	295
5.6.5	The Number e and the Function $\exp x$ Revisited	297
5.6.6	Oscillations	300
5.6.7	Problems and Exercises	303
5.7	Primitives	307
5.7.1	The Primitive and the Indefinite Integral	307
5.7.2	The Basic General Methods of Finding a Primitive	309
5.7.3	Primitives of Rational Functions	315
5.7.4	Primitives of the Form $\int R(\cos x, \sin x) dx$	319
5.7.5	Primitives of the Form $\int R(x, y(x)) dx$	321
5.7.6	Problems and Exercises	324
6	Integration	329
6.1	Definition of the Integral	329
6.1.1	The Problem and Introductory Considerations	329
6.1.2	Definition of the Riemann Integral	331

XVI Table of Contents

6.1.3	The Set of Integrable Functions	333
6.1.4	Problems and Exercises	345
6.2	Linearity, Additivity and Monotonicity of the Integral	347
6.2.1	The Integral as a Linear Function on the Space $\mathcal{R}[a, b]$	347
6.2.2	The Integral as an Additive Interval Function	347
6.2.3	Estimation, Monotonicity, the Mean-value Theorem	350
6.2.4	Problems and Exercises	358
6.3	The Integral and the Derivative	359
6.3.1	The Integral and the Primitive	359
6.3.2	The Newton–Leibniz Formula	361
6.3.3	Integration by Parts and Taylor’s Formula	362
6.3.4	Change of Variable in an Integral	364
6.3.5	Some Examples	367
6.3.6	Problems and Exercises	371
6.4	Some Applications of Integration	374
6.4.1	Additive Interval Functions and the Integral	374
6.4.2	Arc Length	377
6.4.3	The Area of a Curvilinear Trapezoid	383
6.4.4	Volume of a Solid of Revolution	384
6.4.5	Work and Energy	385
6.4.6	Problems and Exercises	391
6.5	Improper Integrals	393
6.5.1	Definition, Examples, and Basic Properties	393
6.5.2	Convergence of an Improper Integral	398
6.5.3	Improper Integrals with More than one Singularity	405
6.5.4	Problems and Exercises	408
7	Functions of Several Variables	411
7.1	The Space \mathbb{R}^m and its Subsets	411
7.1.1	The Set \mathbb{R}^m and the Distance in it	411
7.1.2	Open and Closed Sets in \mathbb{R}^m	413
7.1.3	Compact Sets in \mathbb{R}^m	415
7.1.4	Problems and Exercises	417
7.2	Limits and Continuity of Functions of Several Variables	418
7.2.1	The Limit of a Function	418
7.2.2	Continuity of a Function of Several Variables	423
7.2.3	Problems and Exercises	428
8	Differential Calculus in Several Variables	429
8.1	The Linear Structure on \mathbb{R}^m	429
8.1.1	\mathbb{R}^m as a Vector Space	429
8.1.2	Linear Transformations $L : \mathbb{R}^m \rightarrow \mathbb{R}^n$	430
8.1.3	The Norm in \mathbb{R}^m	431
8.1.4	The Euclidean Structure on \mathbb{R}^m	433

8.2	The Differential of a Function of Several Variables	434
8.2.1	Differentiability and the Differential at a Point	434
8.2.2	Partial Derivatives of a Real-valued Function	435
8.2.3	Coordinate Representation. Jacobians	438
8.2.4	Partial Derivatives and Differentiability at a Point	439
8.3	The Basic Laws of Differentiation	440
8.3.1	Linearity of the Operation of Differentiation	440
8.3.2	Differentiation of a Composite Mapping (Chain Rule) .	442
8.3.3	Differentiation of an Inverse Mapping	448
8.3.4	Problems and Exercises	449
8.4	Real-valued Functions of Several Variables	455
8.4.1	The Mean-value Theorem	455
8.4.2	A Sufficient Condition for Differentiability	457
8.4.3	Higher-order Partial Derivatives	458
8.4.4	Taylor's Formula	461
8.4.5	Extrema of Functions of Several Variables	463
8.4.6	Some Geometric Images	470
8.4.7	Problems and Exercises	474
8.5	The Implicit Function Theorem	480
8.5.1	Preliminary Considerations	480
8.5.2	An Elementary Implicit Function Theorem	482
8.5.3	Transition to a Relation $F(x^1, \dots, x^m, y) = 0$	486
8.5.4	The Implicit Function Theorem	489
8.5.5	Problems and Exercises	494
8.6	Some Corollaries of the Implicit Function Theorem	498
8.6.1	The Inverse Function Theorem	498
8.6.2	Local Reduction to Canonical Form	503
8.6.3	Functional Dependence	508
8.6.4	Local Resolution of a Diffeomorphism	509
8.6.5	Morse's Lemma	512
8.6.6	Problems and Exercises	515
8.7	Surfaces in \mathbb{R}^n and Constrained Extrema	517
8.7.1	k -Dimensional Surfaces in \mathbb{R}^n	517
8.7.2	The Tangent Space	522
8.7.3	Extrema with Constraint	527
8.7.4	Problems and Exercises	540
Some Problems from the Midterm Examinations		545
1.	Introduction to Analysis (Numbers, Functions, Limits)	545
2.	One-variable Differential Calculus	546
3.	Integration. Introduction to Several Variables	547
4.	Differential Calculus of Several Variables	549

XVIII Table of Contents

Examination Topics	551
1. First Semester	551
1.1. Introduction and One-variable Differential Calculus	551
2. Second Semester	553
2.1. Integration. Multivariable Differential Calculus	553
References	557
1. Classic Works	557
1.1 Primary Sources	557
1.2. Major Comprehensive Expository Works	557
1.3. Classical courses of analysis from the first half of the twentieth century	557
2. Textbooks	558
3. Classroom Materials	558
4. Further Reading	559
Subject Index	561
Name Index	573