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Günter M. Ziegler

Lectures on Polytopes

Updated Seventh Printing of the First Edition

Springer

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Preface

The aim of this book is to introduce the reader to the fascinating world of convex polytopes. The book developed from a course that I taught at the Technische Universität Berlin, as a part of the Graduierten-Kolleg "Algorithmische Diskrete Mathematik." I have tried to preserve some of the flavor of lecture notes, and I have made absolutely no effort to hide my enthusiasm for the mathematics presented, hoping that this will be enough of an excuse for being "informal" at times.

There is no P2C2E in this book.*

Each of the ten lectures (or chapters, if you wish) ends with extra notes and historical comments, and with exercises of varying difficulty, among them a number of open problems (marked with an asterisk*), which I hope many people will find challenging. In addition, there are lots of pointers to interesting recent work, research problems, and related material that may sidetrack the reader or lecturer, and are intended to do so.

Although these are notes from a two-hour, one-semester course, they have been expanded so much that they will easily support a four-hour course. The lectures (after the basics in Lectures 0 to 3) are essentially independent from each other. Thus, there is material for quite different two-hour courses in this book, such as a course on "duality, oriented matroids, and zonotopes" (Lectures 6 and 7), or one on "polytopes and polyhedral complexes" (Lectures 4, 5 and 9), etc.

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*P2C2E = "Process too complicated to explain" [469]

Still, I have to make a disclaimer. Current research on polytopes is very much alive, treating a great variety of different questions and topics. Therefore, I have made no attempt to be encyclopedic in any sense, although the notes and references might appear to be closer to this than the text. The main pointers to current research in the field of polytopes are the book by Grünbaum (in its new edition [252]) and the handbook chapters by Klee & Kleinschmidt [329] and by Bayer & Lee [63].

To illustrate that behind all of this mathematics (some of it spectacularly beautiful) there are REAL PEOPLE, I have attempted to compile a bibliography with REAL NAMES (i.e., including first names). In the few cases where I couldn't find more than initials, just assume that's all they have (just like T. S. Garp).

In fact, the masters of polytope theory are really nice and supportive people, and I want to thank them for all their help and encouragement with this project. In particular, thanks to Anders Björner, Therese Biedl, Lou Billera, Jürgen Eckhoff, Eli Goodman, Martin Henk, Richard Hotzel, Peter Kleinschmidt, Horst Martini, Peter McMullen, Ricky Pollack, Jörg Rambau, Jürgen Richter-Gebert, Hans Scheuermann, Tom Shermer, Andreas Schulz, Oded Schramm, Mechthild Stoer, Bernd Sturmfels, and many others for their encouragement, comments, hints, corrections, and references. Thanks especially to Gil Kalai, for the possibility of presenting some of his wonderful mathematics. In particular, in Section 3.4 we reproduce his paper [299],

- GIL KALAI:
  A simple way to tell a simple polytope from its graph,
  J. Combinatorial Theory Ser. A **49** (1988), 381–383;
  ©1988 by Academic Press Inc.,

with kind permission of Academic Press.

My typesetting relies on LaTeX; the drawings were done with `xfig`. They may not be perfect, but I hope they are clear. My goal was to have a drawing on (nearly) every page, as I would have them on a blackboard, in order to illustrate that this really is geometry.

Thanks to everybody at ZIB and to Martin Grötschel for their continuing support.

Berlin, July 2, 1994
Günter M. Ziegler

Preface to the Second Printing

At the occasion of the second printing I took the opportunity to make some revisions, corrections and updates, to add new references, and to report about some very recent work.

However, as with the original edition there is no claim or even attempt to be complete or encyclopedic. I can offer only my own, personal selection. So, I could include only some highlights from and pointers to Jürgen Richter-Gebert's new book [459], which provides substantial new insights about 4-polytopes, and solved a number of open problems from the first version of this book, including all the problems that I had posed in [574]. A summary of some recent progress on polytopes is [576].

Also after this revision I will try to update this book in terms of an electronic preprint "Updates, Corrections, and More," the latest and hottest version of which you should always be able to get at

`http://www.math.tu-berlin.de/~ziegler`

Your contributions to this update are more than welcome.

For the first edition I failed to include thanks to Winnie T. Pooh for his support during this project. I wish to thank Therese Biedl, Joe Bonin, Gabor Hetyei, Winfried Hochstättler, Markus Kiderlen, Victor Klee, Elke Pose, Jürgen Pulkus, Jürgen Richter-Gebert, Raimund Seidel, and in particular Günter Rote for useful comments and corrections that made it into this revised version. Thanks to Torsten Heldmann for everything.

Berlin, June 6, 1997
Günter M. Ziegler

Preface to the Seventh Printing

It is wonderful to see that the “Lectures on Polytopes” are widely used as a textbook in Discrete Geometry, as an introduction to the combinatorial theory of polytopes, and as a starting point for fascinating research.

Thus, resisting for the moment a temptation to “rewrite” and expand the book, I have done a lot of small updates on the text while leaving the general format (and the page numbering) intact. In particular, I have updated the bibliography, and added quite a number of new references, many of them referring to open problems in the original 1995 edition of this book that have in the meantime been fiercefully attacked — and at least partially solved.

In Lecture 0, some examples are given for explicit computations of polytopes that I did using the PORTA software system [151]. It is wonderful that by now we have a much more powerful and comprehensive system for the computation and combinatorial analysis of polytopes, the POLYMAKE system by Michael Joswig and Ewgenij Gawrilow [225, 226, 227]. Use it!

There are two new references available now that I would like to point you to: Jiří Matoušek’s “Lectures on Discrete Geometry” [382], and the second edition of Branko Grünbaum’s classic “Convex Polytopes” [252], which I had already announced in the 1995 preface to this book, and which finally appeared in 2003 — a complete reprint of the book plus more than 100 pages of notes, updates, and new references. Grünbaum received the 2005 AMS Steele Prize for Exposition for his book, which very deservedly marks its importance as the book that created the theory of polytopes as we know it and to a large part guided its development until today.

On the occasion of this new revised printing, I want to thank my Springer editors Tom von Förster, Joachim Heinze, Ina Lindemann, and most recently Ann Kostant for their support over the years.

Finally, of the many other persons that I am grateful to and would like to thank on this occasion let me name only one: Torsten Heldmann.

Berlin, March 19, 2007
Günter M. Ziegler

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