

Series on Advances in Mathematics for Applied Sciences – Vol. 68

DIFFERENTIAL EQUATIONS, BIFURCATIONS, AND CHAOS IN ECONOMICS

Wei-Bin Zhang

Ritsumeikan Asia Pacific University, Japan

 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

DIFFERENTIAL EQUATIONS, BIFURCATIONS, AND CHAOS IN ECONOMICS
Series on Advances in Mathematics for Applied Sciences — Vol. 68

Copyright © 2005 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN 981-256-333-4

Printed in Singapore by B & JO Enterprise

Contents

1. Differential Equations in Economics	1
1.1 Differential Equations and Economic Analysis	3
1.2 Overview	8
Part I: Dimension One	
2. Scalar Linear Differential Equations	16
2.1 Scalar Linear First-Order Differential Equations	18
2.2 A Few Special Types	26
2.3 Second-Order Linear Differential Equations	36
2.4 Higher-Order Linear Differential Equations	41
2.5 Higher-Order Linear Differential Equations with Constant Coefficients	45
3. Scalar Nonlinear Differential Equations	53
3.1 Nonlinear Differential Equations	55
3.2 Stability of Equilibrium of Autonomous Equations	63
3.3 Bifurcations	67
3.4 Periodic Solutions	80
3.5 The Energy Balance Method and Periodic Solutions	84
3.6 Estimation of Amplitude and Frequency	88
4. Economic Dynamics with Scalar Differential Equations	92
4.1 The One-Sector Growth (OSG) Model	93
4.2 The OSG Model with the Cobb-Douglas Production Function	102
4.3 The OSG Model with General Utility Functions	105
4.4 Urban Growth with Housing Production	111
4.5 Endogenous Time in the OSG Model	122

4.6	The OSG Model with Sexual Division of Labor and Consumption	130
4.7	The Uzawa Two-Sector Model	140
4.8	Refitting the Uzawa Model within the OSG Framework	142
 Part II: Dimension Two		
5.	Planar Linear Differential Equations	156
5.1	Planar Linear First-Order Homogeneous Differential Equations	156
5.2	Some Concepts for Qualitative Study	162
5.3	Matrix Exponentials and Reduction to Canonical Forms	164
5.4	Topological Equivalence in Planar Linear Systems	167
5.5	Planar Linear First-Order Nonhomogeneous Differential Equations	171
5.6	Constant-Coefficients Nonhomogeneous Linear Equations with Time-Dependent Terms	178
6.	Planar Nonlinear Differential Equations	182
6.1	Local Stability and Linearization	182
6.2	Liapunov Functions	199
6.3	Bifurcations in Planar Dynamical Systems	211
6.4	Periodic Solutions and Limit Cycles	216
6.5	The Poincaré-Bendixson Theorem	224
6.6	Lienard Systems	229
6.7	The Andronov-Hopf Bifurcations in Planar Systems	233
7.	Planar Dynamical Economical Systems	241
7.1	The IS-LM Model	242
7.2	An Optimal Foreign Debt Model	247
7.3	The Simplified Keynesian Business Cycle Model	250
7.4	The Welfare Economy with Unemployment	255
7.5	Regional Growth with Endogenous Time Distribution	266
7.6	Growth with International Trade and Urban Pattern Formation	275
7.7	A Dynamic Macro Model with Monetary Policy	286
7.8	Economic Growth with Public Services	290
7.9	Endogenous Population Growth in the Ramsey Framework	293
7.10	The Ramsey Model with Endogenous Time	298
 Part III: Higher Dimensions		
8.	Higher-Dimensional Differential Equations	304
8.1	Systems of Linear Differential Equations	304
8.2	Homogeneous Linear Systems with Constant Coefficients	313

8.3	Higher-Order Equations	318
8.4	Diagonalization	322
8.5	The Fundamental Theorem for Linear Systems	325
9.	Higher-Dimensional Nonlinear Differential Equations	329
9.1	Local Stability and Linearization	329
9.2	Liapunov Functions	334
9.3	Conservative Systems	340
9.4	Poincaré Maps	348
9.5	Center Manifold Theorems	354
9.6	Applying the Center Manifold Theorem and the Liapunov Theorem to a Simple Planar System	363
9.7	The Hopf Bifurcation Theorem and Its Applications	367
9.8	The Lorenz Equations and Chaos	374
10.	Higher-Dimensional Economic Evolution	382
10.1	Tâtonnement Price Adjustment Processes	383
10.2	The Three-Country Trade Model with Capital Accumulation	389
10.3	Growth, Trade, and Wealth Distribution Among Groups	395
10.4	A Two-Region Growth Model with Capital and Knowledge	401
10.5	Money and Economic Growth	412
10.6	Limit Cycles and Aperiodic Behavior in the Multi-Sector Optimal Growth Model	418
10.7	Theoretical Insight into China's Modern Economic Development	423
11.	Epilogue: Economic Evolution with Changeable Speeds and Structures	446
	Appendix	455
A.1	Matrix Theory	455
A.2	Systems of Linear Equations	459
A.3	Properties of Functions and the Implicit Function Theorem	460
A.4	Taylor Expansion and Linearization	464
A.5	Structural Stability	466
A.6	Optimal Control Theory	467
	Bibliography	475
	<i>Index</i>	483