Encyclopaedia of Mathematical Sciences

Volume 86

Editor-in-Chief: R.V. Gamkrelidze

Springer-Verlag Berlin Heidelberg GmbH

M. I. Zelikin

Control Theory and Optimization I

Homogeneous Spaces and the Riccati Equation in the Calculus of Variations

Springer

Title of the Russian edition:
Odnorodnye Prostranstva i uravnenie rikkati v variatsionnom ischislenii
Publisher Faktorial, Moscow 1998

Mathematics Subject Classification (1991): 49-XX, 53-XX

ISSN 0938-0396

ISBN 978-3-642-08603-8 ISBN 978-3-662-04136-9 (eBook)
DOI 10.1007/978-3-662-04136-9

Originally published by Springer-Verlag Berlin Heidelberg New York in 2000
Softcover reprint of the hardcover 1st edition 2000

Typesetting: By B. Everett using a Springer TEX macro package.
Printed on acid-free paper SPIN 10732015 46/3143LK - 5 4 3 2 1 0

List of Editors, Authors and Translators

Editor-in-Chief

R.V. Gamkrelidze, Russian Academy of Sciences, Steklov Mathematical Institute, ul. Gubkina 8, 117966 Moscow; Institute for Scientific Information (VINITI), ul. Usievicha 20a, 125219 Moscow, Russia; e-mail: gam@ipsun.ras.ru

Author

M. I. Zelikin, Department of Mathematics, MGU, Vorob'evy Gory, 119899 Moscow, Russia

Translator

S. A. Vakhrameev, Department of Mathematics, VINITI, ul. Usievicha 20a, 125219 Moscow, Russia

Preface

This book is devoted to the development of geometric methods for studying and revealing geometric aspects of the theory of differential equations with quadratic right-hand sides (Riccati-type equations), which are closely related to the calculus of variations and optimal control theory.

The book contains the following three parts, to each of which a separate book could be devoted:

1. the classical calculus of variations and the geometric theory of the Riccati equation (Chaps. 1–5),
2. complex Riccati equations as flows on Cartan–Siegel homogeneity domains (Chap. 6), and
3. the minimization problem for multiple integrals and Riccati partial differential equations (Chaps. 7 and 8).

Chapters 1–4 are mainly auxiliary. To make the presentation complete and self-contained, I here review the standard facts (needed in what follows) from the calculus of variations, Lie groups and algebras, and the geometry of Grassmann and Lagrange–Grassmann manifolds. When choosing these facts, I prefer to present not the most general but the simplest assertions. Moreover, I try to organize the presentation so that it is not obscured by formal and technical details and, at the same time, is sufficiently precise.

Other chapters contain my results concerning the matrix double ratio, complex Riccati equations, and also the Riccati partial differential equation, which arises in the minimization problem for a multiple integral.

The book is based on a course of lectures given in the Department of Mechanics and Mathematics of Moscow State University during several years. Therefore, when writing the book, I imagined the ideal readers to be the undergraduate and graduate students in this department, who are familiar with the foundations of calculus, differential equations, differential geometry, and algebra (although, in some cases, I assumed that the reader is familiar with a deeper mathematical technique). However, I hope that a wider audience will find this book interesting. I also hope that the informed reader will tolerate aspects that seem trivial to him while the reader unfamiliar with one or another mathematical object and encountering some difficulties in understanding the text will resolve them using the literature cited and then excuse me for the less-detailed explanation. Always in such cases, an author should strive for a balance between the difficult and the obvious in order to transform the first into the second. The reader can conclude whether I am successful in this.

I am grateful to my friends and colleagues for their indispensible help in preparing this book. I am particularly grateful to the Candidates of Physics and Mathematics V. F. Borisov, A. V. Dombrin, and L. F. Zelikina, to my student R. Hildebrant for the laborious work in improving the text, to Professors A. V. Arutyunov, A. S. Mishchenko, A. N. Parshin, and E. L. Tonkov for their very useful advice and suggestions, to Professor G. Freiling for very valuable bibliographic information, to Professor S. S. Demidov for his valuable help in the history of mathematics, and to E. Yu. Khodan for the careful editing of the manuscript.

The publication of the Russian edition of this book would have been impossible without the financial support of the Russian Foundation for Basic Research (Grant No. 95-01-02867 for publication of the book, Grant No. 96-01-01360 for research, and Grant No. 96-15-96072 for supporting leading scientific schools), for which I am grateful.

I am deeply grateful to Professor R. Gamkrelidze, Professor S. Vakhrameev, B. Everett, and Springer-Verlag for providing the publication of the English translation of this book.

M. I. Zelikin Moscow, September 1999

Contents