Eberhard Zeidler

Applied Functional Analysis Main Principles and Their Applications

With 37 Illustrations



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