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Commutative Algebra

Volume II

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PREFACE

This second volume of our treatise on commutative algebra deals largely with three basic topics, which go beyond the more or less classical material of volume I and are on the whole of a more advanced nature and a more recent vintage. These topics are: (a) valuation theory; (b) theory of polynomial and power series rings (including generalizations to graded rings and modules); (c) local algebra. Because most of these topics have either their source or their best motivation in algebraic geometry, the algebro-geometric connections and applications of the purely algebraic material are constantly stressed and abundantly scattered throughout the exposition. Thus, this volume can be used in part as an introduction to some basic concepts and the arithmetic foundations of algebraic geometry. The reader who is not immediately concerned with geometric applications may omit the algebro-geometric material in a first reading (see "Instructions to the reader," page vii), but it is only fair to say that many a reader will find it more instructive to find out immediately what is the geometric motivation behind the purely algebraic material of this volume.

The first 8 sections of Chapter VI (including § 5bis) deal directly with properties of places, rather than with those of the valuation associated with a place. These, therefore, are properties of valuations in which the value group of the valuation is not involved. The very concept of a valuation is only introduced for the first time in § 8, and, from that point on, the more subtle properties of valuations which are related to the value group come to the fore. These are illustrated by numerous examples, taken largely from the theory of algebraic function fields (§§ 14, 15). The last two sections of the chapter contain a general treatment, within the framework of arbitrary commutative integral domains, of two concepts which are of considerable importance in algebraic geometry (the Riemann surface of a field and the notions of normal and derived normal models).

The greater part of Chapter VII is devoted to classical properties of polynomial and power series rings (e.g., dimension theory) and their applications to algebraic geometry. This chapter also includes a treatment of graded rings and modules and such topics as characteristic (Hilbert) functions and chains of syzygies. In the past, these last two topics represented some final words of the algebraic theory, to be followed only by deeper geometric applications. With the modern development of homological methods in commutative algebra, these topics became starting points of extensive, purely algebraic theories, having a much wider range of applications. We could not include, without completely disrupting the balance of this volume, the results which require the use of truly homological methods (e.g., torsion and extension functors, complexes, spectral sequences). However, we have tried to include the results which may be proved by methods which, although inspired by homological algebra, are nevertheless classical in nature. The reader will find these results in Chapter VII, §§ 12 and 13, and in Appendices 6 and 7. No previous knowledge of homological algebra is needed for reading these parts of the volume. The reader who wants to see how truly homological methods may be applied to commutative algebra is referred to the original papers of M. Auslander, D. Buchsbaum, A. Grothendieck, D. Rees, J.-P. Serre, etc., to a forthcoming book of D. C. Northcott, as well, of course, as to the basic treatise of Cartan-Eilenberg.

Chapter VIII deals with the theory of local rings. This theory provides the algebraic basis for the local study of algebraic and analytical varieties. The first six sections are rather elementary and deal with more general rings than local rings. Deeper results are presented in the rest of the chapter, but we have not attempted to give an encyclopedic account of the subject.

While much of the material appears here for the first time in book form, there is also a good deal of material which is new and represents current or unpublished research. The appendices treat special topics of current interest (the first 5 were written by the senior author; the last two by the junior author), except that Appendix 6 gives a smooth treatment of two important theorems proved in the text. Appendices 4 and 5 are of particular interest from an algebro-geometric point of view.

We have not attempted to trace the origin of the various proofs in this volume. Some of these proofs, especially in the appendices, are new. Others are transcriptions or arrangements of proofs taken from original papers.

We wish to acknowledge the assistance which we have received from M. Hironaka, T. Knapp, S. Shatz, and M. Schlesinger in the work of checking parts of the manuscript and of reading the galley proofs. Many improvements have resulted from their assistance.

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INSTRUCTIONS TO THE READER

As this volume contains a number of topics which either are of somewhat specialized nature (but still belong to pure algebra) or belong to algebraic geometry, the reader who wishes first to acquaint himself with the basic algebraic topics before turning his attention to deeper and more specialized results or to geometric applications, may very well skip some parts of this volume during a first reading. The material which may thus be postponed to a second reading is the following:

CHAPTER VI

All of § 3, except for the proof of the first two assertions of Theorem 3 and the definition of the rank of a place; § 5: Theorem 10, the lemma and its corollary; § 5bis (if not immediately interested in geometric applications); § 11: Lemma 4 and pages 57-67 (beginning with part (b) of Theorem 19); § 12; § 14: The last part of the section, beginning with Theorem 34′; § 15 (if not interested in examples); §§ 16, 17, and 18.

CHAPTER VII

§§ 3, 4, 4bis, 5 and 6 (if not immediately interested in geometric applications); all of § 8, except for the statement of Macaulay's theorem and (if it sounds interesting) the proof (another proof, based on local algebra, may be found in Appendix 6); § 9: Theorem 29 and the proof of Theorem 30 (this theorem is contained in Theorem 25); § 11 (the contents of this section are particularly useful in geometric applications).

CHAPTER VIII

All of § 5, except for Theorem 13 and its Corollary 2; § 10; § 11: Everything concerning multiplicities; all of § 12, except for Theorem 27 (second proof recommended) and the statement of the theorem of Cohen-Macaulay; § 13.

All appendices may be omitted in a first reading.

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