MEMOIRS of the American Mathematical Society

Number 793

Descriptive Set Theory and Definable Forcing

Jindřich Zapletal



January 2004 • Volume 167 • Number 793 (third of 5 numbers) • ISSN 0065-9266

American Mathematical Society Providence, Rhode Island

Contents

•

1	Intr	oduction		1
	1.1	The subject of the book		. 1
	1.2	The structure of the book		. 2
	1.3	History and acknowledgments		
	1.4	Notation and literature		
2	Defi	inable forcing adding a single real		5
	2.1	The factor algebras		. 5
	2.2	Basic descriptive set theoretic considerations		. 9
	2.3	Examples		
		2.3.1 The ideal of countable sets		
		2.3.2 The ideal of σ -bounded sets		
		2.3.3 The ideal of meager sets		
		2.3.4 The c_{\min} ideal \ldots \ldots \ldots \ldots		
		2.3.5 Ideals generated by closed sets		
		2.3.6 The Laver ideal		
		2.3.7 Ideals associated with creature forcings		
		2.3.8 The Lebesgue null ideal		
		2.3.9 Mathias forcing		4
		2.3.10 The E_0 ideal \ldots \ldots \ldots \ldots \ldots		
		2.3.11 Silver forcing		
		2.3.12 The σ -porous ideal		
		2.3.13 Steprans forcing		
		2.3.14 Hausdorff measures		
		2.3.15 Unions of ideals		
		2.3.16 Cross-products of ideals		
		2.3.17 The σ -splitting ideal		
		2.3.18 Namba forcing		
3	The	countable support iteration		47
	3.1	A topological view of the iteration		
	3.2	The iterated Fubini powers of an ideal		
	3.3	A dichotomy for Π_1^1 on Σ_1^1 ideals		
	3.4	A dichotomy for almost full ideals		
		a ~		

٠

	3.5	Other dichotomies	62
	3.6	Cardinal invariants of the iterated ideals	66
4	\mathbf{Oth}	er forcings	71
	4.1		71
			72
		4.1.2 The ideals associated with countable length iterations	74
		4.1.3 The properties of the factor ordering	77
		4.1.4 The uncountable length	80
		4.1.5 Sacks forcing iteration	81
	4.2	Towers of ideals	82
		4.2.1 Shooting a club with no infinite subset in the ground model	84
		4.2.2 Shooting a club with finite intersection with every ground	
		model ordertype ω set \ldots \ldots \ldots \ldots \ldots	86
5			91
	5.1	Ciesielski-Pawlikowski Axiom CPA and variations	91
		5.1.1 The axioms	91
		5.1.2 Absoluteness with no large cardinals	94
		5.1.3 Absoluteness with large cardinals	97
	5.2	0	.00
	5.3	▲	.02
	5.4		.06
		0	.06
		0 1	.09
		5.4.3 Uniformity of σ -ideal generated by closed sets 1	.14
Δ	Eva	mples of cardinal invariants 1	17
	LAU		
В	The		19
	B.1	The covering numbers 1	.19
	B.2		.23
C	TF.	ative dependentions and the sum	25
С	Elle	ctive descriptive set theory 1	20
D	Lar	ge cardinals 1	33
			34

8