

**Applied Mathematical Sciences | Volume 14**

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# Stability Theory and the Existence of Periodic Solutions and Almost Periodic Solutions



Springer-Verlag New York · Heidelberg · Berlin  
1975

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AMS Classifications: 34D20, 34C25

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*Library of Congress Cataloging in Publication Data*

Yoshizawa, Taro, 1919-

Stability theory and the existence of periodic solutions and almost periodic solutions.

(Applied mathematical sciences; v. 14)

Bibliography: p.

Includes index.

1. Differential equations—Numerical solutions.

2. Stability. 3. Almost periodic functions.

I. Title. II. Series.

QA1.A647 vol. 14 [QA372] 510'.8s [515'.35] 74-28140

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ISBN-13: 978-0-387-90112-1

e-ISBN-13: 978-1-4612-6376-0

DOI: 10.1007/978-1-4612-6376-0

## PREFACE

Since there are several excellent books on stability theory, the author selected some recent topics in stability theory which are related to existence theorems for periodic solutions and for almost periodic solutions. The author hopes that these notes will also serve as an introduction to stability theory. These notes contain stability theory by Liapunov's second method and somewhat extended discussion of stability properties in almost periodic systems, and the existence of a periodic solution in a periodic system is discussed in connection with the boundedness of solutions, and the existence of an almost periodic solution in an almost periodic system is considered in connection with some stability property of a bounded solution. In the theory of almost periodic systems, one has to consider almost periodic functions depending on parameters, but most of text books on almost periodic functions do not contain this case. Therefore, as mathematical preliminaries, the first chapter is intended to provide a guide for some properties of almost periodic functions with parameters as well as for properties of asymptotically almost periodic functions.

These notes originate from a seminar on stability theory given by the author at the Mathematics Department of Michigan State University during the academic year 1972-1973. The author is very grateful to Professor Pui-Kei Wong and members of the Department for their warm hospitality and many helpful conversations. The author wishes to thank Mrs. Katherine MacDougall for her excellent preparation of the text. The author is also indebted to Professor Junji Kato for his helpful criticisms of the manuscript and to Professor Shui-Nee Chow for his careful proofreading of this material.

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