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Jianhong Wu

Theory and Applications of Partial Functional Differential Equations

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To My Parents

Preface

Partial functional differential equations arise from many biological, chemical, and physical systems which are characterized by both spatial and temporal variables and exhibit various spatio-temporal patterns. The systematic study of such equations from the dynamical systems and semigroups point of view began in the 70s, and considerable advances have been achieved since then.

However, most existing results on partial functional differential equations are scattered throughout the research journals, and these results have been obtained by using various methods, concepts, and results from semigroup theory, dynamical system theory, linear and nonlinear functional analysis, ordinary and partial differential equations, and functional differential equations. This makes it difficult for a newcomer to enter into this important research field.

The aim of this book is to provide an introduction to the qualitative theory and applications of semilinear partial functional differential equations from the dynamical systems point of view. The book will focus on those important techniques which have been developed and utilized to generalize various results from ordinary (functional, partial) differential equations to semilinear partial functional differential equations. Special emphasis will be on reaction diffusion equations with delays in the nonlinear reaction terms, and a considerable part of this book will be devoted to the demonstration of how the interaction of spatial diffusion and time delay gives rise to complicated dynamical behaviors and spatio-temporal patterns. Various applications will be provided to motivate and to demonstrate general theory.

I have attempted to make the book as self-contained as possible by including some preliminary materials on semigroups and partial differential equations. Thus the prerequisites are at the level of a graduate student. I hope that my style of presentation will be appealing to people trained and interested in qualitative theory of ordinary and functional differential equations.

It should be emphasized that the choice of material is highly subjective. In particular, this book does not treat reaction diffusion equations with delay in those terms containing spatial partial derivatives. Also, the already lengthy list of references is by no means exhaustive, and I apologize for the exclusion of many other related works.

I would like to acknowledge the encouragement and advice of Professor Jack Hale and Glen Webb, and I am deeply indebted to two scientific advisors of this series for their support and many valuable comments which lead to signifiicant improvements. Special thanks are due to Professor Leon Glass and Professor Raymond Kapral for their helpful suggestions and comments. I am also greatful to the editors and production staff at Springer-Verlag for their assistance.

I wish to express my great appreciation to Professor Joseph So for initially inspiring me to write this book, for his generious help and for his careful reading of the earlier version of the book. Professor Herb Freedman, Professor M. Muldoon, and Professor Shigui Ruan also read various portions of the manuscript and offered many helpful comments which are greatly appreciated. I would like to express my gratitute to Professor Lynn Erbe, Professor Herb Freedman, Professor John Haddock, Professor Senlin Li and Professor Zhicheng Wang for teaching me the theory and applications of differential equations and dynamical systems and for being constant sources of inspiration and encouragement.

It is a great pleasure to acknowledge the financial support from the Natural Sciences and Engineering Research Council of Canada. I would also like to thank the Faculty of Arts (York University) for supporting this project by awarding me a fellowship during 1993-94.

Finally, I would like to thank my wife Ming for her understanding and support during the production of this book.

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