Applied Mathematical Sciences | Volume 50

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(continued on inside back cover)

Calvin H. Wilcox

Sound Propagation in Stratified Fluids

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Preface

This monograph was begun during my sabbatical year in 1980, when I was a visiting professor at the University of Bonn, and completed at the University of Utah in 1981. Preliminary studies were carried out during the period 1972-79 at Utah and while I held visiting professorships at the University of Liège (1973-74), the University of Stuttgart (1974 and 1976-77) and the Ecole Polytechnique Fédérale of Lausanne (1979). Throughout this period my research was supported by the U.S. Office of Naval Research. I should like to express here my appreciation for the support of the Universities of Bonn, Liège, Stuttgart and Utah, the Ecole Polytechnique Fédérale, the Alexander von Humboldt Foundation and the Office of Naval Research which made the work possible. My special thanks are expressed to Professors S. Chatterji (Lausanne), H. G. Garnir (Liège), R. Leis (Bonn) and P. Werner (Stuttgart) for arranging my visits to their universities. I also want to thank Professor Jean Claude Guillot of the University of Paris who collaborated with me during the period 1975-78 on the spectral theory of the Epstein operator. That work, and our many discussions during that period of wave propagation in stratified media, contributed importantly to the final form of the work presented here.

Calvin H. Wilcox
Bonn
August, 1982

Introduction

Stratified fluids whose densities, sound speeds and other parameters are functions of a single depth coordinate occur widely in nature. Indeed, the earth's gravitational field imposes a stratification on its atmosphere, oceans and lakes. It is well known that their stratification has a profound effect on the propagation of sound in these fluids. The most striking effect is probably the occurrence of acoustic ducts, due to minima of the sound speed, that can trap sound waves and cause them to propagate horizontally. The reflection, transmission and distortion of sonar signals by acoustic ducts is important in interpreting sonar echoes. Signal scattering by layers of microscopic marine organisms is important to both sonar engineers and marine biologists. Again, reflection of signals from bottom sediment layers overlying a penetrable bottom are of interest both as sources of unwanted echoes and in the acoustic probing of such layers. Many other examples could be given.

The purpose of this monograph is to develop from first principles a theory of sound propagation in stratified fluids whose densities and sound speeds are essentially arbitrary functions of the depth. In physical terms, the propagation of both time-harmonic and transient fields is analyzed. The corresponding mathematical model leads to the study of boundary value problems for a scalar wave equation whose coefficients contain the prescribed density and sound speed functions. In the formalism adopted here these problems are intimately related to the spectral analysis of a partial differential operator, acting in a Hilbert space of functions defined in the domain occupied by the fluid.

The intended audience for this monograph includes both those applied physicists and engineers who are concerned with sound propagation in stratified fluids and those mathematicians who are interested in spectral analysis and boundary value problems for partial differential operators.

An attempt to address simultaneously two such disparate groups must raise the question: is there a common domain of discourse? The honest answer to this question is no! Current mathematical literature on spectral analysis and boundary value problems is based squarely on functional analysis, particularly the theory of linear transformations in Hilbert spaces. This theory has been readily accessible ever since the publication of M. H. Stone's AMS Colloquium volume in 1932. Nevertheless, the theory has not become a part of the curricula of applied physics and engineering and it is seldom seen in applied science literature on wave propagation. Instead, that literature is characterized by, on the one hand, the use of heuristic non-rigorous arguments and, on the other, by formal manipulations that typically involve divergent series and integrals, generalized functions of unspecified types and the like.

The differences in style and method outlined above pose a dilemma. Can an exposition of our subject be written that is accessible and useful to both applied scientists and mathematicians? An attempt is made to do this below by beginning each chapter with a substantial summary. Taken together, the summaries present the basic physical concepts and results of the theory, formulated in the simplest and most concise form consistent with their nature.

The purpose of the summaries is twofold. First, they can be interpreted in the heuristic way favored by applied physicists and engineers. When read in this way they are independent of the rest of the text and present a complete statement of the physical content of the theory. Second, readers conversant with Hilbert space theory can interpret the summaries as concise statements of the principal concepts and results of the rigorous mathematical theory. When read in this second way, the summaries serve as an introduction to and overview of the complete theory.

Contents