Applied Mathematical Sciences

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Calvin H. Wilcox

Scattering Theory for Diffraction Gratings

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Preface

The scattering of acoustic and electromagnetic waves by periodic surfaces plays a role in many areas of applied physics and engineering. Optical diffraction gratings date from the nineteenth century and are still widely used by spectroscopists. More recently, diffraction gratings have been used as coupling devices for optical waveguides. Trains of surface waves on the oceans are natural diffraction gratings which influence the scattering of electromagnetic waves and underwater sound. Similarly, the surface of a crystal acts as a diffraction grating for the scattering of atomic beams. This list of natural and artificial diffraction gratings could easily be extended.

The purpose of this monograph is to develop from first principles a theory of the scattering of acoustic and electromagnetic waves by periodic surfaces. In physical terms, the scattering of both time-harmonic and transient fields is analyzed. The corresponding mathematical model leads to the study of boundary value problems for the Helmholtz and d'Alembert wave equations in plane domains bounded by periodic curves. In the formalism adopted here these problems are intimately related to the spectral analysis of the Laplace operator, acting in a Hilbert space of functions defined in the domain adjacent to the grating.

The intended audience for this monograph includes both those applied physicists and engineers who are concerned with diffraction gratings and those mathematicians who are interested in spectral analysis and scattering theory for partial differential operators. An attempt to address simultaneously two such disparate groups must raise the question: is there a common domain of discourse? The honest answer to this question is no! Current mathematical literature on spectral analysis and scattering theory is based squarely on functional analysis, particularly the theory of linear transformations in Hilbert spaces. This theory has been readily accessible ever since the publication of M. H. Stone's AMS Colloquium volume in 1932. Nevertheless, the theory has not become a part of the curricula of applied physics and engineering and it is seldom seen in applied science literature on wave propagation and scattering. Instead, that literature is characterized by, on the one hand, the use of heuristic non-rigorous arguments and, on the other, by formal manipulations that typically involve divergent series and integrals, generalized functions of unspecified types and the like.

The differences in style and method outlined above pose a dilemma. Can an exposition of our subject be written that is accessible and useful to both applied scientists and mathematicians? An attempt is made to do this below by dividing the work into two parts. Part 1, called Physical Theory, presents the basic physical concepts and results, formulated in the simplest and most concise form consistent with their nature. Moreover, Part 1 can be interpreted in two different ways. First, it can be interpreted in the heuristic way favored by applied physicists and engineers. When read in this way it presents a complete statement of the physical content of the theory. Second, for readers conversant with Hilbert space theory Part 1 can be interpreted as a concise statement of the principal concepts and results of a rigorous mathematical theory.

When read in the second way Part 1 serves as an introduction to and overview of the complete theory which is presented in Part 2, Mathematical Theory. This part develops the relevant concepts and results from functional analysis and the theory of partial differential equations and applies them to give complete proofs of the results formulated in Part 1. At the same time many secondary concepts and results are formulated and proved that lead to a deeper understanding of the nature and limitations of the theory.

Acknowledgments

Preliminary studies for this work began in 1974, while I was a visiting professor at the University of Stuttgart, and continued there during my tenure as an Alexander von Humboldt Foundation Senior Scientist in 1976-77. The work was completed during my sabbatical year in 1980 when I was a visiting professor at the University of Bonn with the support of the Sonderforschungsbereich 72. Throughout this period my research was supported by the U. S. Office of Naval Research. I should like to express here my appreciation for the support of the Universities of Bonn, Stuttgart and Utah, the Alexander von Humboldt Foundation and the Office of Naval Research which made the work possible. My special thanks are expressed to Professor Rolf Leis, University of Bonn, and Professor Peter Werner, University of Stuttgart, for arranging my visits to their universities.

I should also like to thank here Professor Jean Claude Guillot of the University of Paris for helpful discussions in 1977-78 of a preliminary version of this work. Finally, and most important of all, I want to express my thanks to Dr. Hans Dieter Alber of the University of Bonn for his outstanding paper of 1979 on steady-state scattering by periodic surfaces. It was the concepts introduced in this paper that led to the final, very general, theory developed here. Dr. Alber's contributions have influenced nearly every part of this work.

Calvin H. Wilcox
Bonn
July, 1982

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