

Stephen Wiggins

Introduction to Applied Nonlinear Dynamical Systems and Chaos

With 291 Illustrations



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Stephen Wiggins
Department of Applied Mechanics
California Institute of Technology
Pasadena, California 91125, USA

Series Editors

F. John
Courant Institute of
Mathematical Sciences
New York University
New York, NY 10012
USA

J.E. Marsden
Department of Mathematics
University of California
Berkeley, CA 94720
USA

L. Sirovich
Division of Applied
Mathematics
Brown University
Providence, RI 02912
USA

M. Golubitsky
Department of
Mathematics
University of Houston
Houston, TX 77004
USA

W. Jäger
Department of Applied
Mathematics
Universität Heidelberg
Im Neuenheimer Feld 294
6900 Heidelberg, FRG

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