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Fractals and Hyperspaces

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The picture on the front cover shows a zoom-in on Fig. 14, page 51

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Foreword

The main theme of this monograph is the study of fractals and fractal notions, backed up by a self-contained nonstandard development of relevant hyperspace theory, particularly as regards the Hausdorff metric and Vietoris topology. The fractal study itself is in two parts, the first developing and making contributions to the theory of J. E. Hutchinson's invariant sets, sets which are self-similar in the sense of being composed of smaller images of themselves. The second part explores newer territory, introducing the formal notion of a 'view' as part of a general framework concerned with studying the structure and perception of sets within a given space, and in particular we use views to express and investigate new concepts of self-similarity and fractality which are then considered in connection with invariant sets, a large class of which are shown to be 'visually fractal' in a certain precise sense. Complete with many figures and suggestions for further work, the monograph should be of relevance to those interested in fractals, hyperspaces, fixed-point theory, tilings, or nonstandard analysis.

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Keith R. Wicks
Hull, August 1991.

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