A FIRST GRADUATE COURSE IN ABSTRACT ALGEBRA

W. J. Wickless
University of Connecticut
Storrs, Connecticut, U.S.A.

0 04 040 0 0 0 0



MARCEL DEKKER, INC.

New York · Basel

This edition published in the Taylor & Francis e-Library, 2009.

To purchase your own copy of this or any of Taylor & Francis or Routledge's collection of thousands of eBooks please go to www.eBookstore.tandf.co.uk.

Although great care has been taken to provide accurate and current information, neither the author(s) nor the publisher, nor anyone else associated with this publication, shall be liable for any loss, damage, or liability directly or indirectly caused or alleged to be caused by this book. The material contained herein is not intended to provide specific advice or recommendations for any specific situation.

Trademark notice: Product or corporate names may be trademarks or registered trademarks and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress.

Headquarters

Marcel Dekker, Inc., 270 Madison Avenue, New York, NY 10016, U.S.A. tel: 212–696–9000; fax: 212–685–4540

Distribution and Customer Service

Marcel Dekker, Inc., Cimarron Road, Monticello, New York 12701, U.S.A. tel: 800–228–1160; fax: 845–796–1772

Eastern Hemisphere Distribution

Marcel Dekker AG, Hutgasse 4, Postfach 812, CH-4001 Basel, Switzerland tel: 41-61-260-6300; fax: 41-61-260-6333

World Wide Web

http://www.dekker.com

The publisher offers discounts on this book when ordered in bulk quantities. For more information, write to Special Sales/Professional Marketing at the headquarters address above.

Copyright © 2004 by Marcel Dekker, Inc.

All Rights Reserved.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage and retrieval system, without permission in writing from the publisher.

ISBN 0-203-91366-3 Master e-book ISBN

ISBN: 0-8247-5627-4 (Print Edition)

Contents

Preface	XVII
Part 1	
Chapter 1 Groups (mostly finite)	
1.1 Definitions, examples, elementary properties	1
1.2 Subgroups, cyclic groups	
1.3 Factorization in Z	8
First Problem Set	9
1.4 Isomorphism	10
Second Problem Set	11
1.5 Homomorphisms	11
Third Problem Set	13
1.6 Normal subgroups and factor groups	13
Fourth Problem Set	17
1.7 Simple groups and composition series	18
Fifth Problem Set	20
1.8 Symmetric groups	20
Sixth Problem Set	25
1.9 Conjugacy classes, p-groups, solvable groups	25
Seventh Problem Set	28
1.10 Direct products	28
Eighth Problem Set	31
1.11 Sylow theorems	31
Ninth Problem Set	36
1.12 The structure of finite abelian groups	36
Tenth Problem Set	40

xx Contents

Chapter 2	Rings	(mostly	domains)	
-----------	-------	---------	----------	--

2.1	Definitions and elementary properties	41
	Eleventh Problem Set	44
2.2	Homomorphisms, ideals and factor rings	45
	Twelfth Problem Set	48
2.3	Principal ideal domains	48
	Thirteenth Problem Set	51
2.4	Polynomials	51
	Fourteenth Problem Set	56
2.5	I[x] is a ufd*	56
	Fifteenth Problem Set	57
2.6	Euclidean domains*	58
	Sixteenth Problem Set	60
Chapter 3 Modul	les	
3.1	Elementary concepts	62
	Seventeenth Problem Set	64
3.2	Free and projective modules	64
	Eighteenth Problem Set	67
3.3	Tensor products	67
	Nineteenth Problem Set	72
3.4	Finitely generated modules over a pid	72
	Twentieth Problem Set	74
3.5	A structure theorem	74
	Twenty-first Problem Set	78
Chapter 4 Vector	Spaces	
4.1	Definitions and glossary	80
4.2	Time for a little set theory	80
4.3	A structure theorem for vector spaces	83
	Twenty-second Problem Set	86
4.4	Finite remarks on finite dimensional vector spaces	86
	Twenty-third Problem Set	88
4.5	Matrices and systems of equations	88
	Twenty-fourth Problem Set	92
4.6	Linear transformations and matrices	92
	Twenty-fifth Problem Set	95 Masemile protetto de copyright

4.7	Determinants	96
	Twenty-sixth Problem Set	102
4.8	Characteristic values, vectors, basis change	102
	Twenty-seventh Problem Set	107
4.9	Canonical forms	108
	Twenty-eighth Problem Set	114
4.10	Dual spaces*	114
	Twenty-ninth Problem Set	115
4.11	Inner product spaces*	116
	Thirtieth Problem Set	118
4.12	Linear functionals and adjoints*	119
	Thirty-first Problem Set	122
Part 2		
Chapter 5 Fields	and Galois theory	
5.1	Preliminary results	124
	Thirty-second Problem Set	125
5.2	Straight edge and compass construction	126
	Thirty-third Problem Set	128
5.3	Splitting fields	128
	Thirty-fourth Problem Set	133
5.4	The algebraic closure of a field*	133
	Thirty-fifth Problem Set	135
5.5	A structure theorem for finite fields	135
	Thirty-sixth Problem Set	135
5.6	The Galois correspondence	136
	Thirty-seventh Problem Set	137
5.7	Galois criterion for radical solvability	144
	Thirty-eighth Problem Set	149
5.8	The general equation of degree <i>n</i> Thirty-ninth Problem Set	150 151

Contents xxi

xxii Contents

Chapter 6 Topics	s in noncommutative rings	
6.1	Introduction	152
6.2	Simple modules	152
	Fortieth Problem Set	153
6.3	The Jacobson Radical	153
	Forty-first Problem Set	156
6.4	The Jacobson Density Theorem	156
6.5	Semisimple Artinian rings	158
	Forty-second Problem Set	161
6.6	Structure of complex group algebras	162
6.7	Applications to finite groups	164
	Forty-third Problem Set	165
Chapter 7 Group	extensions	
7.1	Introduction	166
7.2	Exact sequences and ZG-modules	166
	Forty-fourth Problem Set	169
7.3	Semidirect products	169
	Forty-fifth Problem Set	171
7.4	Extensions and factor sets	171
	Forty-sixth Problem Set	173
7.5	Solution of the extension problem	174
	Forty-seventh Problem Set	177
Chapter 8 Topics	s in abelian groups	
8.1	Direct sums and products	179
	Forty-eighth Problem Set	179
8.2	Structure theorem for divisible groups	180
	Forty-ninth Problem Set	182
8.3	Rank one torsion-free groups	183
	Fiftieth Problem Set	184
8.4	Structure of completely decomposable groups	185
	Fifty-first Problem Set	186
8.4	Algebraically compact groups	186
	Fifty-second Problem Set	190

		Contents	XXII
8.5	Structure of algebraically compact groups		191
	Fifty-third Problem Set		194
8.6	Structure of countable torsion groups		194
	Fifty-fourth Problem Set		199
Refere	ences		200
Index			201