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Differential and Integral Inequalities

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Preface

In 1964 the author's monograph "Differential- und Integral-Ungleichungen," with the subtitle "und ihre Anwendung bei Abschätzungs- und Eindeutigkeitsproblemen" was published. The present volume grew out of the response to the demand for an English translation of this book. In the meantime the literature on differential and integral inequalities increased greatly. We have tried to incorporate new results as far as possible. As a matter of fact, the Bibliography has been almost doubled in size.

The most substantial additions are in the field of existence theory. In Chapter I we have included the basic theorems on Volterra integral equations in Banach space (covering the case of ordinary differential equations in Banach space). Corresponding theorems on differential inequalities have been added in Chapter II. This was done with a view to the new sections, dealing with the line method, in the chapter on parabolic differential equations. Section 35 contains an exposition of this method in connection with estimation and convergence. An existence theory for the general nonlinear parabolic equation in one space variable based on the line method is given in Section 36. This theory is considered by the author as one of the most significant recent applications of inequality methods. We should mention that an exposition of Krzyżański's method for solving the Cauchy problem has also been added.

The numerous requests that the new edition include a chapter on elliptic differential equations have been satisfied to some extent. A survey of the most important theorems on elliptic differential inequalities, with brief proofs, is given in an appendix. We note that all new material has been incorporated in such a way as to leave unchanged the numbers of sections, subsections, formulas, . . . appearing in the German edition.

I am most grateful to the translator, Lisa Rosenblatt, for her reliable collaboration. My thanks also go to Professor L. F. Shampine for his valuable assistance in resolving technical difficulties. I am greatly indebted to Professor B. Noble, Professor G. H. Knightly and Mr. Wickwire, who read parts of the manuscript and made valuable suggestions, and to Dr. H. Becker, Dr. K. Deimling, Dr. E. Mues, Dr. G.

Schleinkofer and Mr. A. Voigt for their assistance in proof-reading. Finally, the publisher, who expedited the completion of this work in every way with patience in the face of many problems, deserves my special thanks.

Karlsruhe, August 1970 Wolfgang Walter

Preface to the German Edition

The theory of differential and integral inequalities has been greatly enriched in the past fifteen years by new understanding and knowledge. This book is the first comprehensive presentation of these developments. It covers Volterra integral equations (in one and several variables) as well as ordinary, hyperbolic, and parabolic differential equations.

The present volume of "Springer Tracts" is simultaneously textbook, guide to the literature, and research monograph. My intention of writing a self-contained textbook that can also be read by advanced students was consistent with the subject. The theory of differential and integral inequalities is basically of an elementary nature and no special preparation is needed for its understanding. Relatively much effort is spent on one-dimensional problems in the first two chapters to show the methods to be used. Thus the essential ideas of the proofs are first clearly worked out in very elementary cases. The central core of the book is basically concerned with partial differential equations, and particularly with parabolic equations, to which the last chapter, by far the most extensive, is devoted.

My special thanks are due to Professor K. Nickel for many fruitful discussions and suggestions. His advice was indispensable in writing the section on boundary layer theory, in which I see the most important and most elegant application of the theory to date.

My gratitude also goes to Dr. H. Brakhage, Dr. P. Werner, and Mr. H. Weigel for their valuable help in proofreading and for their criticisms.

The Editor, Professor L. Collatz, furthered the progress of this work by his continued interest during the entire time of its preparation. I wish to thank him, as well as Springer-Verlag, for their willing cooperation and the excellent production of this book.

Karlsruhe, March 1964 Wolfgang Walter

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Notation

Formulas are numbered with Arabic numerals; theorems, definitions, remarks, etc., are given Roman numbers in sequence. Thus 8 X (α) means hypothesis (α) of Theorem X of Section 8, (27.2) means formula (2) of Section 27. References within a section omit the section number. Literature references include the author and the publication date in parentheses; if this is ambiguous, an identifying letter is added, e.g., Ciliberto (1956 b).

The following notational principles will be used (see also the list of notations at the end of the bool). Independent variables are denoted by $t, \tau \in E^1$ and $x = (x_1, \ldots, x_m)$, $\xi = (\xi_1, \ldots, \xi_m) \in E^m$ (not bold-face), while $z \in E^1$ and $\mathbf{z} = (z_1, \ldots, z_n) \in E^n$ (bold-face) denote a function and a system of functions, respectively; n is the number of equations in a system of differential equations or integral equations. The function classes $\mathscr{D}, \mathscr{E}, \mathscr{H}, \mathscr{P}$ refer to the "right hand sides" of differential equations or kernels of integral equations, which are generally denoted by f, k, or ω. The domain of definition of a function f is denoted by $D(f)$; $D(f)$ can have a very different meaning depending, for example, on whether $f = f(t, z)$ or $f = f(t, x, z, p, r)$. The classes $Z, Z_0, Z_c, \ldots$ ("admissible" functions) refer to solutions or approximate solutions of the problems under consideration. These symbols also have different meanings in different chapters, yet are consistent. Thus, for instance, for ordinary differential equations Z is the class of functions $\varphi(t)$ which are continuous for $0 \leqq t \leqq T$ and differentiable for $0 < t \leqq T$, while for parabolic differential equations Z stands for the class of functions $\varphi = \varphi(t, x)$ described in detail in 23 III. Yet for a function $\varphi = \varphi(t)$ which is independent of x, Definitions 5 I and 23 III are equivalent. For the most part, no assumptions will be made concerning the domain of definition $D(f)$ of the right hand side f of a differential equation. The function classes $Z(f), Z_0(f), \ldots$ take this into account. If, for example, an ordinary differential equation $u' = f(t, u)$ is given for $0 < t \leqq T$, then $u \in Z(f)$ means, first, that u is in the class Z defined above and, second, that u "can be substituted" in f, i.e., $(t, u(t)) \in D(f)$ for $0 < t \leqq T$.

The concept of monotonicity (and of quasimonotonicity) is defined in 6 II in the weak sense, i.e., with equality permitted. If equality is excluded, we speak of strict monotonicity. Thus a real-valued function