W.D. Wallis

A *Beginner's* Guide to Graph Theory

Springer Science+Business Media, LLC

W.D. Wallis
Department of Mathematics
Southern Illinois University
Carbondale, IL 62901
U.S.A.

Library of Congress Cataloging-in-Publication Data

Wallis, W.D.
A beginner's guide to graph theory / W.D. Wallis.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-4757-3136-1 ISBN 978-1-4757-3134-7 (eBook)
DOI 10.1007/978-1-4757-3134-7

1. Graph Theory. I. Title.

QA166.W314 2000
511'.5–dc21 00-031172
CIP

Math Subject Classifications 2000: 05-01, 05Cxx

Printed on acid-free paper
©2000 Springer Science+Business Media New York
Originally published by Birkhäuser Boston in 2000

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC.
except for brief excerpts in connection with reviews or
scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.
The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

ISBN 978-1-4757-3136-1 SPIN 10758558

Typeset by the author in LaTeX.
Cover design by Jeff Cosloy, Newton, MA.

9 8 7 6 5 4 3 2 1

Contents

Preface vii

List of Figures xv

1 Graphs 1
1.1 Sets, Binary Relations and Graphs 1
1.2 Some Definitions 4
1.3 Degree 11

2 Walks, Paths and Cycles 15
2.1 Basic Ideas 15
2.2 Weights and Shortest Paths 19
2.3 Euler Walks 23
2.4 Hamilton Cycles 26
2.5 The Traveling Salesman Problem 31

3 Cuts and Connectivity 35
3.1 Cutpoints and Bridges 35
3.2 Blocks 37
3.3 Connectivity 40

4 Trees 43
4.1 Characterizations of Trees 43
4.2 Spanning Trees 46
4.3 Minimal Spanning Trees 51

5 Linear Spaces Associated with Graphs **55**
5.1 Finite Fields and Vector Spaces 55
5.2 The Power Set as a Vector Space 56
5.3 The Vector Spaces Associated with a Graph 58
5.4 The Cutset Subspace 60
5.5 Bases and Spanning Trees 63

6 Factorizations **69**
6.1 One-Factorizations 69
6.2 Tournament Applications of One-Factorizations 75
6.3 A General Existence Theorem 77
6.4 Graphs Without One-Factors 81

7 Graph Colorings **85**
7.1 Vertex Colorings 85
7.2 Brooks' Theorem 89
7.3 Counting Vertex Colorings 91
7.4 Edge-Colorings 96
7.5 Class 2 Graphs 99

8 Planarity **105**
8.1 Representations and Crossings 105
8.2 Euler's Formula 108
8.3 Maps, Graphs and Planarity 111

9 Ramsey Theory **115**
9.1 The Graphical Case of Ramsey's Theorem 115
9.2 Ramsey Multiplicity 120
9.3 Application of Sum-Free Sets 123
9.4 Bounds on Classical Ramsey Numbers 125
9.5 The General Case of Ramsey's Theorem 129

10 Digraphs **131**
10.1 Basic Ideas 131
10.2 Orientations and Tournaments 135
10.3 Directed Euler Walks 139

11 Critical Paths **143**
11.1 Activity Digraphs 143
11.2 Critical Path Analysis 146
11.3 Critical Paths Under Uncertainty 153

12 Flows in Networks **159**
12.1 Transportation Networks and Flows 159
12.2 Maximal Flows 165
12.3 The Max Flow Min Cut Theorem 171

12.4 The Max Flow Min Cut Algorithm 173
12.5 Supply and Demand Problems 179

13 Computational Considerations **185**
13.1 Computation Time 185
13.2 Data Structures 188
13.3 Some Graph Algorithms 190
13.4 Intractability 194

References **197**

Hints **205**

Answers and Solutions **207**

ndex **225**