Graduate Texts in Mathematics 145

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S. Axler F.W. Gehring K.A. Ribet

Springer Science+Business Media, LLC

Graduate Texts in Mathematics

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60 ARNOLD. Mathematical Methods in Classical Mechanics. 2nd ed.
61 WHITEHEAD. Elements of Homotopy Theory.

(continued after index)

James W. Vick

Homology Theory

An Introduction to Algebraic Topology

Second Edition

With 78 Illustrations

Springer

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Mathematics Subject Classifications (1991): 55-01, 55N10, 57M10

Library of Congress Cataloging-in-Publication Data
Vick, James W.
Homology theory: an introduction to algebraic topology 2nd ed / James W. Vick.
p. cm.—(Graduate texts in mathematics)
Originally published: New York: Academic Press, c1973. (Pure and
applied mathematics; v. 53).
Includes bibliographical references and index.
ISBN 978-1-4612-6933-5 ISBN 978-1-4612-0881-5 (eBook)
DOI 10.1007/978-1-4612-0881-5
(Berlin: alk. paper)
1. Homology theory. I. Title. II. Series.
QA612.3.V53 1994
514'.23—dc20 93-5255

Printed on acid-free paper.

The first edition of this book was published by Academic Press, New York, © 1973.

© 1994 Springer Science+Business Media New York
Originally published by Springer-Verlag Berlin Heidelberg New York in 1994
Softcover reprint of the hardcover 2nd edition 1994
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Production coordinated by TechEdit Production Services and managed by Natalie Johnson;
manufacturing supervised by Genieve Shaw.
Typeset by Asco Trade Typesetting Ltd., Hong Kong.

9 8 7 6 5 4 3 2

ISBN 978-1-4612-6933-5 SPIN 10750186

to Niki, Todd, and Stuart

Preface to the Second Edition

The 20 years since the publication of this book have been an era of continuing growth and development in the field of algebraic topology. New generations of young mathematicians have been trained, and classical problems have been solved, particularly through the application of geometry and knot theory. Diverse new resources for introductory coursework have appeared, but there is persistent interest in an intuitive treatment of the basic ideas.

This second edition has been expanded through the addition of a chapter on covering spaces. By analysis of the lifting problem it introduces the fundamental group and explores its properties, including Van Kampen's Theorem and the relationship with the first homology group. It has been inserted after the third chapter since it uses some definitions and results included prior to that point. However, much of the material is directly accessible from the same background as Chapter 1, so there would be some flexibility in how these topics are integrated into a course.

The Bibliography has been supplemented by the addition of selected books and historical articles that have appeared since 1973.

Preface to the First Edition

During the past twenty-five years the field of algebraic topology has experienced a period of phenomenal growth and development. Along with the increasing number of students and researchers in the field and the expanding areas of knowledge have come new applications of the techniques and results of algebraic topology in other branches of mathematics. As a result there has been a growing demand for an introductory course in algebraic topology for students in algebra, geometry, and analysis, as well as for those planning further work in topology.

This book is designed as a text for such a course as well as a source for individual reading and study. Its purpose is to present as clearly and concisely as possible the basic techniques and applications of homology theory. The subject matter includes singular homology theory, attaching spaces and finite CW complexes, cellular homology, the Eilenberg–Steenrod axioms, cohomology, products, and duality and fixed-point theory for topological manifolds. The treatment is highly intuitive with many figures to increase the geometric understanding. Generalities have been avoided whenever it was felt that they might obscure the essential concepts.

Although the prerequisites are limited to basic algebra (abelian groups) and general topology (compact Hausdorff spaces), a number of the classical applications of algebraic topology are given in the first chapter. Rather than devoting an initial chapter to homological algebra, these concepts have been integrated into the text so that the motivation for the constructions is more apparent. Similarly the exercises have been spread throughout in order to exploit techniques or reinforce concepts.

At the close of the book there are three bibliographical lists. The first includes all works referenced in the text. The second is an extensive list of

books and notes in algebraic topology and related fields, and the third is a similar list of survey and expository articles. It was felt that these would best serve the student, teacher, and reader in offering accessible sources for further reading and study.

Acknowledgments

Acknowledgments to the First Edition

The original manuscript for this book was a set of lecture notes from Math 401–402 taught at Princeton University in 1969–1970. However, much of the technique and organization of the first four chapters may be traced to courses in algebraic topology taught by Professor E.E. Floyd at the University of Virginia in 1964–1965 and 1966–1967. The author was one of the fortunate students who have been introduced to the subject by such a masterful teacher. Any compliments that this book may merit should justifiably be directed first to Professor Floyd.

The author wishes to express his appreciation to the students and faculty of Princeton University and the University of Texas who have taken an interest in these notes, contributed to their improvement, and encouraged their publication. The typing of the manuscript by the secretarial staff of the Mathematics Department at the University of Texas was excellent, and particular thanks are due to Diane Schade, who types the majority of it. Many helpful improvements and corrections in the original manuscript were suggested by Professor Peter Landweber.

Finally, the author expresses a deep sense of gratitude to his wife and family for their boundless patience and understanding over years during which this book has evolved.

Acknowledgments to the Second Edition

When John Ewing inquired as to my interest in reissuing this book with Springer-Verlag, I was doubly pleased. First, there was interest in making it

available once more since it went out of print some years ago. Second, I would be offered the chance to include new topics that would give it broader appeal. I am grateful to John and to Springer-Verlag for their interest.

The intervening 20 years at the University of Texas have been superb. My colleagues among the faculty, staff, and students have provided much encouragement and support. In particular, I appreciate the opportunity to work jointly on research with John Alexander, Gary Hamrick, and Pierre Conner.

The essential reason for these happy years is conveyed in the dedication: Niki, Todd, and Stuart. From kindergarten through graduate school, from little league through weddings, and from professional success back to a doctoral program, there have been enough great memories to last a lifetime.

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