

Richard S. Varga

# Matrix Iterative Analysis

Second Revised and Expanded Edition

With 42 Figures

 Springer

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ISSN 0179-3632  
ISBN 978-3-540-66321-8 (hardcover) e-ISBN 978-3-642-05156-2  
ISBN 978-3-642-05154-8 (softcover)  
DOI 10.1007/978-3-642-05156-2  
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2009942152

Mathematics Subject Classification (1991): 15A60, 15A48, 15A06, 41A21, 41A20, 41A10,  
65F10 39A10

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Cover design: SPi Publisher Services

Printed on acid-free paper

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