

Richard S. Varga

Geršgorin and His Circles

 Springer

Richard S. Varga
Kent State University
Institute for Computational Mathematics
Kent, OH 44242, USA
e-mail: varga@mcs.kent.edu

Cover jacket figure:
Wassily Kandinsky, *Heavy Circles*, 1927. Oil on canvas.
Norton Simon Museum,
The Blue Four Galka Scheyer Collection
© VG Bild-Kunst, Bonn 2004

Library of Congress Control Number: 2004104814

Mathematics Subject Classification (2000): 15A18, 15A42, 15A48, 15A60,
65F15, 65F35

ISSN 0179-3632
ISBN 3-540-21100-4 Springer Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable to prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media
springeronline.com

© Springer-Verlag Berlin Heidelberg 2004
Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Cover design: *design&production*, Heidelberg
Typeset by the author using a Springer \LaTeX macro package
Printed on acid-free paper 46/3142LK-543210

Contents

I. Preface	VII
1. Basic Theory	1
1.1 Geršgorin's Theorem	1
1.2 Extensions of Geršgorin's Theorem via Graph Theory	10
1.3 Analysis Extensions of Geršgorin's Theorem and Fan's Theorem	18
1.4 A Norm Derivation of Geršgorin's Theorem 1.1	26
2. Geršgorin-Type Eigenvalue Inclusion Theorems	35
2.1 Brauer's Ovals of Cassini	35
2.2 Higher-Order Lemniscates	43
2.3 Comparison of the Brauer Sets and the Brualdi Sets	53
2.4 The Sharpness of Brualdi Lemniscate Sets	58
2.5 An Example	67
3. More Eigenvalue Inclusion Results	73
3.1 The Parodi-Schneider Eigenvalue Inclusion Sets	73
3.2 The Field of Values of a Matrix	79
3.3 Newer Eigenvalue Inclusion Sets	84
3.4 The Pukhov-Solov'ev Eigenvalue Inclusions Set	92
4. Minimal Geršgorin Sets and Their Sharpness	97
4.1 Minimal Geršgorin Sets	97
4.2 Minimal Geršgorin Sets via Permutations	110
4.3 A Comparison of Minimal Geršgorin Sets and Brualdi Sets ..	121
5. G-Functions	127
5.1 The Sets \mathcal{F}_n and \mathcal{G}_n	127
5.2 Structural Properties of \mathcal{G}_n and \mathcal{G}_n^c	133
5.3 Minimal G-Functions	141
5.4 Minimal G-Functions with Small Domains of Dependence ...	145
5.5 Connections with Brauer Sets and Generalized Brualdi Sets ..	149

6. Geršgorin-Type Theorems for Partitioned Matrices	155
6.1 Partitioned Matrices and Block Diagonal Dominance	155
6.2 A Different Norm Approach	164
6.3 A Variation on a Theme by Brualdi	174
6.4 G-Functions in the Partitioned Case	181
Appendix A. Geršgorin's Paper from 1931, and Comments . .	189
Appendix B. Vector Norms and Induced Operator Norms. . .	199
Appendix C. The Perron-Frobenius Theory of Nonnegative Matrices	201
Appendix D. Matlab 6 Programs.	205
References	217