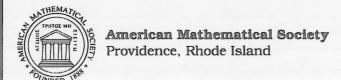
Translations of

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Nontraditional Methods in Mathematical Hydrodynamics

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Contents

Introduction	1
Chapter 1. Stationary Flows of an Ideal Fluid on the Plane §1. Pressure field in a neighborhood of a singular point 1.1. Nondegenerate singular points 1.2. The stream function 1.3. Euler fields §2. Other properties of plane flows 2.1. Equivalent forms of Euler's equations 2.2. Reducible fields 2.3. Frozen-in vortices	11 11 12 12 14 14 15 16 18
2.4. Simple domains 2.5. Holomorphic flows	20
Chapter 2. Topology of Two-Dimensional Flows §3. An extended version of some Morse constructions 3.1. Admissible stream functions 3.2. The basic integer relations 3.3. Graphs and chambers 3.4. The structure of admissible functions §4. Portraits of potential flows §5. Imposition of a vortex on a simple passing flow §6. Imposition of a vortex on an inverse flow	23 23 23 27 30 33 37 46 49
Chapter 3. A Two-Dimensional Passing Flow Problem for Stationary Euler Equations §7. Uniqueness of an analytic solution 7.1. Formulation of the problem	53 53 53 56
7.2. The uniqueness theorem §8. Analysis of the uniqueness of a smooth flow 8.1. The choice of the region of prescribed vorticity 8.2. Uniqueness of an open flow	58 58 58 60
 8.3. Nonuniqueness of a developed flow 8.4. Problems on simple passing and inverse flows §9. Existence theorems 9.1. Auxiliary boundary problem 9.2. The vortex catastrophe 	62 65 65 72
§10. Regular functionals 10.1. The closure of a smooth operator	74

viii CONTENTS

10.2. The conditions for splitting	
10.3. The existence of a critical point	77
10.4. An abstract version of the riversity	80
10.4. An abstract version of the eigenvalue problem	83
Chapter 4. The Dissipative Top and the Navier-Stokes Equations	89
§11. Axioms of the dissipative top	89
11.1. Plane flows of a viscous incompressible fluid	89
11.2. Quasicompact Lie algebras	92
11.3. The equation of the top. A local uniqueness theorem	94
11.4. Types of dissipative tops	97
11.5. The existence and completeness of the system of principal rotations	101
11.6. The basic scalar relations	104
§12. Stability of a principal rotation	105
12.1. A nonlocal uniqueness theorem	105
12.2. A nonlocal theorem on stability	103
§13. Generalized solutions of the equation of the top	112
13.1. Spatial flows of a viscous incompressible fluid	112
13.2. An extension of the class of admissible right-hand sides	116
13.3. The existence of a generalized solution	
§14. Examples of dissipative tops	117
14.1. Motions on the algebra $\mathfrak{so}(n,R)$	123
14.2. Flows on two-dimensional manifolds	123 125
14.3. Flows on a three-dimensional torus	129
14.4. An example of violation of quasicompactness	130
Chapter 5. Specific Features of Turbulence Models §15. Wave properties of turbulent flows	133
15.1. An "ideal" turbulent medium	133
15.2. Field of small perturbations	133
15.3. Characteristics	134
15.4. Effects of viscosity	137
15.5. A proposition on all and the	140
15.5. A proposition on characteristics	146
§16. Passage to turbulent regime and bifurcation 16.1. The closure scheme	151
	151
16.2. An illustrative example	154
16.3. Analysis of the bifurcation problem	154
16.4. The existence of a minimizing element	160
Appendix. Formal Constructions Connected with Euler Equations	163
317. Continuous models of some Lagrangian systems	163
17.1. Notation	163
17.2. Natural motions and the Euler equations	167
17.3. The stationary action principle	170
17.4. Translational movements and motions along geodesics	176
17.5. Equations of relativistic hydrodynamics	177
17.6. The concept of the integrating factor	185
17.7. The invariant form of the Navier-Stokes equations	187
References	
rotototices	101