Introduction to Ramsey Spaces

Stevo Todorcevic

Materiale protetto da copyright

ANNALS OF MATHEMATICS STUDIES

Copyright © 2010 by Princeton University Press

Published by Princeton University Press
41 William Street, Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press
6 Oxford Street, Woodstock, Oxfordshire OX20 1TW

All Rights Reserved

Library of Congress Cataloging-in-Publication Data

Todorcevic, Stevo
Introduction to Ramsey Spaces / Stevo Todorcevic.
p. cm. (Annals of mathematics studies ; no. 174)
Includes bibliographical references and index.
ISBN 978-0-691-14541-9 (hardcover : alk. paper)
ISBN 978-0-691-14542-6 (pbk. : alk. paper)
1. Ramsey theory. 2. Algebraic spaces. I. Title.
QA166.T635 2010
511'.5–dc22 2009036738

British Library Cataloging-in-Publication Data is available

This book has been composed in LaTeX

The publisher would like to acknowledge the author of this volume for providing the camera-ready copy from which this book was printed.

Printed on acid-free paper. ∞

press.princeton.edu

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Materiale protetto da copyright

Contents

Introduction 1

Chapter 1. Ramsey Theory: Preliminaries 3
1.1 Coideals 3
1.2 Dimensions in Ramsey Theory 5
1.3 Higher Dimensions in Ramsey Theory 10
1.4 Ramsey Property and Baire Property 20

Chapter 2. Semigroup Colorings 27
2.1 Idempotents in Compact semigroups 27
2.2 The Galvin-Glazer Theorem 30
2.3 Gowers's Theorem 34
2.4 A Semigroup of Subsymmetric Ultrafilters 38
2.5 The Hales-Jewett Theorem 41
2.6 Partial Semigroup of Located Words 46

Chapter 3. Trees and Products 49
3.1 Versions of the Halpern-Läuchli Theorem 49
3.2 A Proof of the Halpern-Läuchli Theorem 55
3.3 Products of Finite Sets 57

Chapter 4. Abstract Ramsey Theory 63
4.1 Abstract Baire Property 63
4.2 The Abstract Ramsey Theorem 68
4.3 Combinatorial Forcing 76
4.4 The Hales-Jewett Space 83
4.5 Ramsey Spaces of Infinite Block Sequences of Located Words 89

Chapter 5. Topological Ramsey Theory 93
5.1 Topological Ramsey Spaces 93
5.2 Topological Ramsey Spaces of Infinite Block Sequences of Vectors 99
5.3 Topological Ramsey Spaces of Infinite Sequences of Variable Words 105
5.4 Parametrized Versions of Rosenthal Dichotomies 111
5.5 Ramsey Theory of Superperfect Subsets of Polish Spaces 117
5.6 Dual Ramsey Theory 121

Materiale protetto da copyright

5.7 A Ramsey Space of Infinite-Dimensional Vector Subspaces of $F^{\mathbb{N}}$ 127

Chapter 6. Spaces of Trees 135

6.1 A Ramsey Space of Strong Subtrees 135
6.2 Applications of the Ramsey Space of Strong Subtrees 138
6.3 Partition Calculus on Finite Powers of the Countable Dense Linear Ordering 143
6.4 A Ramsey Space of Increasing Sequences of Rationals 149
6.5 Continuous Colorings on $\mathbb{Q}^{[k]}$ 152
6.6 Some Perfect Set Theorems 158
6.7 Analytic Ideals and Points in Compact Sets of the First Baire Class 165

Chapter 7. Local Ramsey Theory 179

7.1 Local Ellentuck Theory 179
7.2 Topological Ultra-Ramsey Spaces 190
7.3 Some Examples of Selective Coideals on $\mathbb{N}$ 194
7.4 Some Applications of Ultra-Ramsey Theory 198
7.5 Local Ramsey Theory and Analytic Topologies on $\mathbb{N}$ 202
7.6 Ultra-Hales-Jewett Spaces 207
7.7 Ultra-Ramsey Spaces of Block Sequences of Located Words 212
7.8 Ultra-Ramsey Space of Infinite Block Sequences of Vectors 215

Chapter 8. Infinite Products of Finite Sets 219

8.1 Semicontinuous Colorings of Infinite Products of Finite Sets 219
8.2 Polarized Ramsey Property 224
8.3 Polarized Partition Calculus 231

Chapter 9. Parametrized Ramsey Theory 237

9.1 Higher Dimensional Ramsey Theorems Parametrized by Infinite Products of Finite Sets 237
9.2 Combinatorial Forcing Parametrized by Infinite Products of Finite Sets 243
9.3 Parametrized Ramsey Property 248
9.4 Infinite-Dimensional Ramsey Theorem Parametrized by Infinite Products of Finite Sets 254

Appendix 259

Bibliography 271

Subject Index 279

Index of Notation 285

Materiale protetto da copyright