

Vladimir V. Tkachuk

# A $C_p$ -Theory Problem Book

Topological and Function Spaces



Springer

Vladimir V. Tkachuk  
Departamento de Matemáticas  
Universidad Autónoma Metropolitana  
Av. San Rafael Atlixco, 186, Col. Vicentina  
Iztapalapa 09340, Mexico City  
Mexico  
vova@xanum.uam.mx

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# Contents

<b>Preface .....</b>	v
<b>1 Basic Notions of Topology and Function Spaces .....</b>	1
1.1 Topologies, Separation Axioms and a Glance at $C_p(X)$ .....	2
1.2 Products, Cardinal Functions and Convergence .....	12
1.3 Metrizability and Completeness .....	24
1.4 Compactness Type Properties in Function Spaces .....	35
1.5 More on Completeness: Realcompact Spaces .....	42
1.6 Bibliographic Notes .....	49
<b>2 Solutions of Problems 001–500 .....</b>	51
<b>3 Bonus Results: Some Hidden Statements .....</b>	435
3.1 Standard Spaces .....	436
3.2 Metrizable Spaces and Compact Spaces .....	437
3.3 Properties of Continuous Maps .....	439
3.4 Covering Properties, Normality and Open Families .....	441
3.5 Product Spaces and Cardinal Invariants .....	443
3.6 Raznoie (Unclassified Results) .....	444
<b>4 Open Problems .....</b>	445
4.1 Local Properties .....	447
4.2 Discreteness of $X$ and Completeness of $C_p(X)$ .....	448
4.3 Dense Subspaces .....	450
4.4 The Lindelöf Property in $X$ and $C_p(X)$ .....	451
4.5 Other Covering Properties .....	453
4.6 Mappings Which Involve $C_p$ -spaces .....	456
4.7 Very General Questions .....	458
4.8 Fuzzy Questions .....	459
4.9 Naïve Questions .....	461
4.10 Raznoie (Unclassified Questions) .....	462

<b>Bibliography</b>	.....	465
<b>List of Special Symbols</b>	.....	475
<b>Index</b>	.....	479