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# A $C_p$ -Theory Problem Book

Special Features of Function Spaces

 Springer

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The author wants to emphasize that if a postgraduate student mastered the material of the first volume, it will be more than sufficient to understand every problem and solution of this book. However, for a concrete topic, much less might be needed. Finally, let me outline some points which show the potential usefulness of the present work:

- *The only background needed is some knowledge of set theory and real numbers; any reasonable course in calculus covers everything needed to understand this book.*
- *The student can learn all of general topology required without recurring to any textbook or papers; the amount of general topology is strictly minimal and is presented in such a way that the student works with the spaces  $C_p(X)$  from the very beginning.*
- *What is said in the previous paragraph is true as well if a mathematician working outside of topology (e.g., in functional analysis) wants to use results or methods of  $C_p$ -theory; he (or she) will find them easily in a concentrated form or with full proofs if there is such a need.*
- *The material we present here is up to date and brings the reader to the frontier of knowledge in a reasonable number of important areas of  $C_p$ -theory.*
- *This book seems to be the first self-contained introduction to  $C_p$ -theory. Although there is an excellent textbook written by Arhangel'skii (1992a), it heavily depends on the reader's good knowledge of general topology.*

Mexico City, Mexico

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