

M. I. Zelikin
V. F. Borisov

Theory of Chattering Control

with applications to
Astronautics, Robotics, Economics, and Engineering

Birkhäuser
Boston • Basel • Berlin

Vladimir Borisov
Department of Mathematics
Moscow Technological Institute
Moscow, 141220
Russia

Michail I. Zelikin
Department of Mathematics
Moscow State University
Moscow, 119899
Russia

Library of Congress Cataloging-in-Publication Data

Borisov, V. F. (Vladimir F.), 1961-

Theory of chattering control with applications to astronautics,
robotics, economics, and engineering / V.F. Borisov, M. I. Zelikin.
p. cm. -- (Systems & control)

Includes bibliographical references and index.

ISBN-13: 978-1-4612-7634-0 e-ISBN-13: 978-1-4612-2702-1

DOI: 10.1007/978-1-4612-2702-1

1. Chattering control (Control systems) 2. Control theory.
I. Zelikin, M. I. (Mikhail Il'ich) II. Title. III. Series
TJ223.C45B67 1994 93-51121
003'.5--dc20 CIP

Printed on acid-free paper

© 1994 Birkhäuser Boston

Softcover reprint of the hardcover 1st edition 1994

Birkhäuser 

Copyright is not claimed for works of U.S. Government employees.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior permission of the copyright owner.

Permission to photocopy for internal or personal use of specific clients is granted by Birkhäuser Boston for libraries and other users registered with the Copyright Clearance Center (CCC), provided that the base fee of \$6.00 per copy, plus \$0.20 per page is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923, U.S.A. Special requests should be addressed directly to Birkhäuser Boston, 675 Massachusetts Avenue, Cambridge, MA 02139, U.S.A.

Typeset by the Authors in AMS-TeX

9 8 7 6 5 4 3 2 1

CONTENTS

Preface	vii
Outline	xv
1. Introduction	1
1.1 The Subject of the Book	1
1.2 Hamiltonian Systems and Singular Extremals	4
1.3 The Semi-Canonical Form of Hamiltonian Systems	9
1.4 Integral Varieties with Chattering Arcs	11
1.5 An Example of Designing a Lagrangian Manifold	13
2. Fuller's Problem	19
2.1 Statement of Fuller's Problem	19
2.2 Chattering Arcs	21
2.3 Untwisted Chattering Arcs	27
2.4 The Geometry of Trajectories of Hamiltonian Systems	29
3. Second Order Singular Extremals and Chattering	38
3.1 Preliminaries	38
3.2 Manifolds with Second Order Singular Trajectories	41
3.3 The Connection with Fuller's Problem	44
3.4 Resolution of the Singularity of the Poincaré Mapping	45
3.5 The Connection with the Problem of C. Marchal	51
3.6 Fixed Points of the Quotient Mapping	53
3.7 The Hyperbolic Structure of the Quotient Mapping	59
3.8 Non-Degeneracy of the Fixed Point	67
3.9 Bundles with Chattering Arcs	68
3.10 Lagrangian Manifolds	72
3.11 Synthesis with Locally Optimal Chattering Arcs	75
3.12 Regular Projection of Chattering Varieties	78
4. The Ubiquity of Fuller's Phenomenon	85
4.1 Kupka's Results	86
4.2 Codimension of the Set of Fuller Points	92
4.3 Structural Stability of the Optimal Synthesis in the Two-Dimensional Fuller Problem	100
5. Higher Order Singular Extremals	105
5.1 Conjectures Concerning Higher Order Singular Modes	105

5.2 Problems with Linear Constraints	110
5.3 Problems with Symmetries	114
5.4 Bi-Constant Ratio Solutions of Fuller's Problems	122
5.5 Optimality of b.c.r. Solutions	130
5.6 Numerical Verification of the Conjecture on the Number of Cycles in the Orbit Space	136
5.7 Three-Dimensional Fuller Problems	138
6. Applications	167
6.1 Fibrations in Three-Dimensional Space	167
6.2 Stabilization of a Rigid Body	171
6.3 The Resource Allocation Problem	178
6.4 Control of Two Interdependent Oscillators	187
6.5 Lowden's Problem	192
6.6 Robot Control	203
7. Multidimensional Control and Chattering Modes	217
7.1 Multidimensional Problems with a Polyhedral Indicatrix	218
7.2 Multidimensional Problems with a Smooth Indicatrix	224
Epilogue	235
List of Figures	236
Bibliography	237
Index	241