

THE NEW STATISTICAL ANALYSIS OF DATA

T.W. Anderson

Stanford University

Jeremy D. Finn

State University of New York at Buffalo



Springer

T.W. Anderson
Stanford University
Department of Statistics
Stanford, CA 94305
USA

Jeremy D. Finn
State University of New York at Buffalo
Graduate School of Education
Buffalo, NY 14260-1000
USA

Library of Congress Cataloging-in-Publication Data

Anderson, T.W. (Theodore Wilbur), 1918–

The new statistical analysis of data / T. W. Anderson, Jeremy D. Finn.

p. cm.

Includes bibliographical references and index.

ISBN-13: 978-1-4612-8466-6

e-ISBN-13: 978-1-4612-4000-6

DOI: 10.1007/978-1-4612-4000-6

1. Statistics. I. Finn, Jeremy D. II. Title. III. Series.

QA276.12.A46 1996

519.5—dc20

95-44885

Printed on acid-free paper.

© 1996 Springer-Verlag New York, Inc.

Softcover reprint of the hardcover 1st edition 1996

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Bill Imbornoni; manufacturing supervised by Joe Quatela.
Typeset by Integre Technical Publishing Co., Inc., Albuquerque, NM.

9 8 7 6 5 4 3

Contents

Preface	vii
Part I: Introduction	1
Chapter 1 — The Nature of Statistics	3
1.1 Some Examples of the Use of Statistics	4
Political Polls, 4 The Polio Vaccine Trial, 9 Smoking and Health, 11	
Project “Head Start”, 13 “A Minority of One Versus a Unanimous	
Majority”, 14 Deciding Authorship, 15	
1.2 Basic Concepts of Statistics	16
Experimental and Nonexperimental Research, 16 Populations and	
Samples, 17 Descriptive and Inferential Statistics, 19 Planning	
Statistical Investigations, 21	
Exercises	23
Part II: Descriptive Statistics	27
Chapter 2 — Organization of Data	29
2.1 Kinds of Variables: Scales	30
Categorical Variables, 30 Numerical Variables, 31 Scales*, 32	
2.2 Organization of Categorical Data	34
Frequencies, 34 Frequency Graphs, 38	
	xiii

2.3	Organization of Numerical Data	40
	Discrete Data, 40 Continuous Data, 46 Frequency Distributions for Continuous Data, 47	
	Exercises	57
Chapter 3 — Measures of Location		69
3.1	The Mode	70
	Definition and Interpretation of the Mode, 70 Mode of Grouped Data, 72	
3.2	The Median and Other Percentiles	74
	The Median, 74 Quartiles, 75 Deciles, Percentiles, and Other Quantiles, 77	
3.3	The Mean	77
	Definition and Interpretation of the Mean, 77 Use of Notation, 79 Calculating the Mean from a Frequency Distribution, 82 The Proportion as a Mean, 85 Other Properties of the Mean, 85 Effects of Change of Scale, 87	
3.4	Choosing Among Measures of Location	88
	Shape of Distributions, 90	
Appendices		93
	Appendix 3A Computing the Median and Other Quantiles of Grouped Continuous Data	93
	Appendix 3B Rules for Summation	95
	Appendix 3C Change of Scale	98
	Appendix 3D Significant Digits	99
	Exercises	99
Chapter 4 — Measures of Variability		107
4.1	Ranges	109
	The Range, 109 The Interquartile Range, 110	
4.2	The Mean Deviation*	111
4.3	The Standard Deviation	112
	Definitions, 112 Reasons for Dividing by One Less than the Sample Size, 114 Interpreting the Standard Deviation, 115	

4.4	Formulas for the Standard Deviation	118
	Computing Formula, 118 Calculating the Standard Deviation from a Frequency Distribution, 119 Effects of Change of Scale, 121	
4.5	Some Uses of Location and Dispersion Measures Together	122
	Standard Scores, 122 Box-and-Whisker Plots, 123	
	Appendices	127
	Appendix 4A Proofs of Some Algebraic Principles	127
	Appendix 4B Adjusting Data to Maintain Computational Accuracy	129
	Exercises	130
Chapter 5 — Summarizing Multivariate Data: Association Between Numerical Scales		137
5.1	Association of Two Numerical Variables	139
	Scatter Plots, 139 Other Information Revealed by Scatter Plots, 144 The Correlation Coefficient, 148 Rank Correlation, 156	
5.2	More than Two Variables	159
	Profiles, 159 Correlation Matrix, 160	
	Appendices	164
	Appendix 5A Computational Form for the Covariance	164
	Appendix 5B Change of Scale	164
	Exercises	166
Chapter 6 — Summarizing Multivariate Data: Association Between Categorical Variables		177
6.1	Two-by-Two Frequency Tables	178
	Organization of Data into Two-by-Two Tables, 178 Calculation of Percentages, 182 Interpretation of Frequencies, 185	
6.2	Larger Two-Way Frequency Tables	193
	Organization of Data for Two Categorical Variables, 193 Interpretation of Frequencies, 195	
6.3	Three Categorical Variables	201
	Organization of Data for Three Yes-No Variables, 201 Larger Three-Way Frequency Tables, 205	

6.4	Effects of a Third Variable	206
	Association and Interpretation, 206 Independence in Subtables, 206	
	Similar Association in Subtables, 211 Reversal of Association in	
	Subtables, 215 Hidden Relationships, 218	
	Exercises	220
Part III: Probability		231
Chapter 7 — Basic Ideas of Probability		233
7.1	Intuitive Examples of Probability	233
	Physical Devices Which Approximate Randomness, 233 The Draft	
	Lottery, 235 Probability and Everyday Life, 236	
7.2	Probability and Statistics	237
7.3	Probability in Terms of Equally Likely Cases	238
7.4	Events and Probabilities in General Terms	242
	Outcomes, Events, and Probabilities, 242 Addition of Probabilities of	
	Mutually Exclusive Events, 243 Addition of Probabilities, 245	
7.5	Interpretation of Probability: Relation to Real Life	247
7.6	Conditional Probability	248
7.7	Independence	252
7.8	Random Sampling; Random Numbers	255
	Random Devices, 255 Random Numbers, 255 Sampling with	
	Replacement, 257 Sampling without Replacement, 258	
7.9	Bayes' Theorem*	258
	Bayes' Theorem in a Simplified Case, 258 Examples, 260 Use of	
	Subjective Prior Probabilities in Bayes' Theorem, 263	
	Exercises	266
Chapter 8 — Probability Distributions		273
8.1	Random Variables	273
8.2	Cumulative Probability	275

8.3	The Mean and Variance of a Probability Distribution	277
	The Mean of a Discrete Random Variable, 277 The Variance of a Discrete Random Variable, 278 The Mean and Variance of a Continuous Random Variable, 279	
8.4	Uniform Distributions	280
	The Discrete Uniform Distribution, 280 The Continuous Uniform Distribution, 281	
8.5	The Family of Normal Distributions	282
	The Normal Distributions, 282 Different Normal Distributions, 284 The Standard Normal Distribution, 285 Other Normal Distributions, 292	
	Exercises	297
Chapter 9—Sampling Distributions		305
9.1	Sampling from a Population	306
	Random Samples, 306 Sampling Distributions, 307 Independence of Random Variables, 309 Sampling from a Probability Distribution, 311	
9.2	Sampling Distributions of a Sum and of a Mean	311
	Sampling Distribution of a Sum, 311 Sampling Distribution of the Sample Mean, 315	
9.3	The Binomial Distribution	316
	Sampling Distribution of the Number of Heads, 316 Proportion of Heads in Bernoulli Trials, 323 The Mean and Variance of the Binomial Probability Distribution, 325	
9.4	The Law of Averages (Law of Large Numbers)*	326
9.5	The Normal Distribution of Sample Means	329
	The Central Limit Theorem, 329 Normal Approximation to the Binomial Distribution, 335	
	Appendix 9A The Correction for Continuity	337
	Exercises	339
Part IV: Statistical Inference		345
Chapter 10—Using a Sample to Estimate Characteristics of One Population		347
10.1	Estimation of a Mean by a Single Number	348

10.2	Estimation of Variance and Standard Deviation	352
10.3	An Interval of Plausible Values for a Mean	353
	Confidence Intervals when the Standard Deviation is Known, 353	
	Confidence Intervals when the Standard Deviation is Estimated, 359	
10.4	Estimation of a Proportion	364
	Point Estimation of a Proportion, 364 Interval Estimation of a Proportion, 365	
10.5	Estimation of a Median	366
	Point Estimation of a Median, 366 Interval Estimation of a Median*, 368	
10.6	Paired Measurements	369
	Mean of a Population of Differences, 369 Matched Samples, 371	
10.7	Importance of Size of Population Relative to Sample Size*	373
	Appendix 10A The Continuity Adjustment	378
	Exercises	379
Chapter 11 — Answering Questions about Population Characteristics		389
11.1	Testing a Hypothesis About a Mean	390
	An Example and Terminology, 391 Hypothesis Testing Procedures, 394 Deciding Whether a Population Mean Differs from a Given Value, 400 Relation of Two-Tailed Tests to Confidence Intervals, 402 Validity Conditions, 404	
11.2	Errors and Power	405
	Types of Error, 405 Probability of a Type I Error, 406 Probability of a Type II Error and Power, 407	
11.3	Testing Hypotheses About a Mean when the Standard Deviation Is Unknown	413
11.4	<i>P</i> Values: Another Way to Report Tests of Significance	419
	<i>P</i> Values for Two-Tailed Tests, 422 <i>P</i> Values when the Population Standard Deviation Is Unknown, 423	
11.5	Testing Hypotheses About a Proportion	424
	Testing Hypotheses About the Probability of a Success, 424 Example of a Two-Tailed Test, 427	
11.6	Testing Hypotheses About a Median: The Sign Test*	429

11.7	Paired Measurements	431
	Testing Hypotheses About the Mean of a Population of Differences, 431	
	Testing the Hypothesis of Equality of Proportions, 436	
	Appendix 11A The Continuity Correction	440
	Exercises	441
Chapter 12 — Differences Between Populations		455
12.1	Comparison of Two Independent Sample Means When the Population Standard Deviations Are Known	458
	One-Tailed Tests, 458 Two-Tailed Tests, 463 Confidence Intervals, 464 Validity Conditions, 467	
12.2	Comparison of Two Independent Sample Means When the Population Standard Deviations are Unknown but Treated as Equal	468
	Confidence Intervals, 473	
12.3	Comparison of Two Independent Sample Means When the Population Standard Deviations Are Unknown and Not Treated as Equal*	474
12.4	Comparison of Two Independent Sample Proportions	477
	Hypothesis Tests, 477 Confidence Intervals, 480	
12.5	The Sign Test for a Difference in Locations*	481
	Appendix 12A The Continuity Adjustment	486
	Exercises	486
Chapter 13 — Variability in One Population and in Two Populations		501
13.1	Variability in One Population	502
	The Sampling Distribution of the Sum of Squared Deviations, 502	
	Testing the Hypothesis that the Variance Equals a Given Number, 506	
	Confidence Intervals for the Variance*, 507	
13.2	Variability in Two Populations	508
	The Sampling Distribution of the Ratio of Two Sample Variances, 508	
	Testing the Hypothesis of Equality of Two Variances, 512 Confidence Intervals for the Ratio of Two Variances*, 515	
	Exercises	516

Part V: Statistical Methods for Other Problems	521
Chapter 14 — Inference on Categorical Data	523
14.1 Tests of Goodness of Fit	524
Two Categories—Dichotomous Data, 524 Any Number of Categories, 526 Combining Categories, 528	
14.2 Chi-Square Tests of Independence	530
Two-by-Two Tables, 530 Two-Way Tables in General, 537 Combining Categories, 540	
14.3 Measures of Association	542
The Phi Coefficient, 542 A Coefficient Based on Prediction, 543 A Coefficient Based on Ordering*, 548	
Appendix 14A The Continuity Adjustment	554
Exercises	554
Chapter 15 — Simple Regression Analysis	569
15.1 Functional Relationship	570
15.2 Statistical Relationship	572
15.3 Least-Squares Estimates	573
15.4 Statistical Inference for β	578
15.5 The Correlation Coefficient: A Measure of Linear Relationship	581
The Bivariate Normal Distribution and Test of Significance for a Correlation, 583	
Exercises	585
Chapter 16 — Comparison of Several Populations	597
16.1 One-Way Analysis of Variance	602
A Complete Example, 602 The Algebra of ANOVA, 607 An Example with Unequal Sample Sizes, 612 More About the Analysis of Variance, 615	
16.2 Which Groups Differ from Which, and by How Much?	619
Comparing Two Means, 619 Comparing More Than Two Means, 622	

16.3	Analysis of Variance of Ranks*	626
	Exercises	629
Chapter 17 — Sampling from Populations: Sample Surveys		639
17.1	Simple Random Sampling	641
17.2	Stratified Random Sampling	644
	Two Strata, 644 More than Two Strata, 648	
17.3	Cluster Sampling	651
17.4	Systematic Sampling with a Random Start	651
17.5	Systematic Subsampling with Random Starts	654
	Exercises	655
Answers to Selected Exercises		659
References		675
Appendices		681
Appendix I	Table of the Standard Normal Distribution	683
Appendix II	Table of Binomial Probabilities	684
Appendix III	Percentage Points of Student's t -Distributions	686
Appendix IV	Percentage Points of Chi-Square Distributions	687
Appendix V	Upper Percentage Points of F -Distributions	688
Appendix VI	Table of Random Digits	701
Index		703