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10

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N. M. Swerdlow
O. Neugebauer

**Mathematical Astronomy
in Copernicus's
De Revolutionibus**

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To
S. Chandrasekhar
with admiration and gratitude

Preface

When I first laid out the framework for *A History of Ancient Mathematical Astronomy*, I intended to carry the discussion down to the last applications of Greek astronomical methodology, i.e. Copernicus, Brahe, and Kepler. But as the work proceeded, it became evident that this plan was much too ambitious, and so I decided to terminate my *History* with late antiquity, well before Islam.

Nevertheless, I did not discard the running commentary that I had prepared when studying *De revolutionibus* in its relation to the methodology of the *Almagest*. Only recently, E. S. Kennedy and his collaborators had opened access to the “Marāgha School” (mainly Ibn ash-Shāṭir), revealing close parallels to Copernicus’s procedures. Accordingly, it seemed useful to make available a modern analysis of *De revolutionibus*, and thus in 1975 I prepared for publication “Notes on Copernicus.”

In the meantime, however, Noel Swerdlow, also starting from Greek astronomy, not only extended his work into a deep analysis of *De revolutionibus*, but also systematically investigated its sources and predecessors (Peurbach, Regiomontanus, etc.). I was aware of these studies through his publications as well as from numerous conversations on the subject at The Institute for Advanced Study and at Brown University. It became clear to me that my own investigations lay at too superficial a level, and I therefore withdrew my manuscript and suggested to Swerdlow that he undertake a thoroughgoing revision and amplification of my “Notes.” His acceptance of my proposal initiated the present publication.

The addition of my name to the title page is nothing more than a reflection of this prehistory. In the present publication Swerdlow has not only substantially extended the investigation of Copernicus’s work in breadth and depth, but has also completely rewritten the whole manuscript. I am happy to see that my own study has thus been replaced by a work of real competence that represents a significant step in our attempts to evaluate the position of ancient science in the evolutionary process of mathematical astronomy.

O. N.

The *Almagest* has been the subject of two recent expositions, by Pedersen and Neugebauer, the appearance of which has made it reasonable to devote similar analyses to other works of mathematical astronomy in the Ptolemaic tradition. The most famous of these is *De revolutionibus*, the last important work to employ the full range of Ptolemaic methods, in its use of heliocentric models showing their nearly universal applicability, and in its failure to go beyond Ptolemy in descriptive and numerical accuracy showing their limitations. Some years ago, I began a rather ambitious project of preparing a translation of *De revolutionibus* with a technical commentary and translation of Peurbach's and Regiomontanus's *Epitome of the Almagest*, Copernicus's principal guide to Ptolemy's astronomy. The appearance of two new translations of *De revolutionibus* in 1976 and 1978, whatever their quality, made a third superfluous, and the *Epitome* deserves a proper edition, translation, and commentary in its own right, for it is not merely ancillary, but, along with *De revolutionibus*, the most important astronomical work of the Renaissance.

However, the commentary still seemed worth pursuing. While a number of papers on special topics have been published in recent years, the last technically proficient exposition of *De revolutionibus* as a whole was a chapter of 40 pages written by Norbert Herz in 1894. Hence, when Neugebauer showed me his original manuscript, it seemed to present the opportunity to undertake an analysis of *De revolutionibus* on about the same scale as the analyses of the *Almagest*. The result has actually ended up somewhat longer than anticipated because of the necessity of explaining the background to Copernicus's work, much of which he reviews himself, from the *Almagest*, the *Epitome*, and a number of other sources, and the decision to set out Copernicus's numerical work in the derivation of parameters—the subject that occupies the greatest part of *De revolutionibus*—reconstructing, as far as possible, the derivations he omits. As the work continued to increase in size, it appeared that it could well end up being more on the subject of Copernicus's mathematical astronomy than anyone cares to know. But it seemed useful to include whatever could reasonably be accomplished within the limits of a general exposition in order to provide a foundation for others who may wish to correct or extend our work without having to start over from the beginning. Even so, the study is far from exhaustive. Certain subjects, such as trigonometry, spherical astronomy, the catalogue of stars, basic properties of eccentric and epicyclic models, to which Copernicus's own contribution is minimal, we have treated briefly, and the first chapters of Book I, which have frequently been commented on at length by scholars whose qualifications far exceed our own, we have omitted entirely. In any case, the study is quite long enough as it is.

It is with gratitude that I acknowledge the support and encouragement of the Department of Astronomy and Astrophysics and the Division of the Physical Sciences of The University of Chicago. They have not only granted me the freedom and best possible conditions for research, but have held

firmly to the principle that historical studies are a proper part of scientific research, and for this I am especially grateful.

There are several people who have helped with this work in a number of ways. Mrs. Maria D'Onofrio of the Sciences Library, Mrs. Madeline Gross-Millin of the Rockefeller Library, and Mrs. Mary Russo of Special Collections of Brown University have always been of the greatest help. My debt to the Department of History of Mathematics of Brown University is not to be measured. Professor Gerald Toomer has read the manuscript of this study, caught any number of errors, and made many suggestions, always correct. Professor David Pingree has always been ready to listen, and answer questions on the most obscure points. The late Professor Abraham Sachs and Mrs. Janet Sachs have watched over me for many years, offering their guidance on many occasions when it was very much needed. And I am especially grateful to Molly Schen, my best and closest friend through four summers in Providence. Of course, my greatest debt is to my co-author, who has taught me whatever I know.

Providence, Summer 1983

N. S.

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