Structure of Decidable Locally Finite Varieties

Ralph McKenzie
Department of Mathematics
University of California
Berkeley, California
U.S.A.

Matthew Valeriote
Department of Mathematics and Statistics
McMaster University
Hamilton, Ontario
Canada

Library of Congress Cataloging-in-Publication Data McKenzie, Ralph,

The structure of decidable locally finite varieties / Ralph McKenzie, Matthew Valeriote.

p. cm. - (Progress in Mathematics; v. 78) Includes bibliographical references.

ISBN 0-8176-3439-8

1. Varieties (Universal algebra) I. Valeriote, Matthew.

II. Title. III. Title: Locally finite varieties. IV. Series:

Progress in mathematics (Boston, Mass.); vol. 78.

QA251.M44 1989

512--dc20

Dr. McKenzie's research is supported by National Science Foundation grant number DMS-8600300.

Dr. Valeriote's research is supported by the Natural Sciences and Engineering Research Council of Canada.

Printed on acid-free paper.

© Birkhäuser Boston, Inc., 1989.

Softcover reprint of the hardcover 1st edition 1989

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without prior permission of the copyright owner.

Permission to photocopy for internal or personal use, or the internal or personal use of specific clients, is granted by Birkhäuser Boston, Inc., for libraries and other users registered with the Copyright Clearance Center (CCC), provided that the base fee of \$0.00 per copy plus \$0.20 per page is paid directly to CCC, 21 Congress Street, Salem, Massachusetts 01970, U.S.A. Special requests should be addressed directly to Birkhäuser Boston, Inc., 675 Massachusetts Avenue, Cambridge, Massachusetts 02139, U.S.A.

3439-8/89 \$0.00 + .20

ISBN-13: 978-1-4612-8908-1 e-ISBN-13: 978-1-4612-4552-0

DOI: 10.1007/978-1-4612-4552-0

Text prepared by the authors in camera-ready form.

Contents

INTRODUC	TION	1	1	
CHAPTER	0:	Preliminaries	5	
	0.1:	Languages, structures, algebras, and graphs	5	
	0.2:	Decidability and interpretability	8	
	0.3:	Varieties	12	
	0.4:	Abelian and solvable algebras	13	
	0.5:	Special kinds of varieties	17	
	0.6: 0.7:	Tame congruence theory Definable relations in subdirect powers	$\frac{20}{27}$	
~				
CHAPTER	1:	Preview: The three subvarieties	35	
PART I: ST	RUCT	TURED VARIETIES		
CHAPTER	2:	A PROPERTY OF THE CENTER	39	
CHAPTER	3:	Centerless algebras	57	
CHAPTER	4:	THE DISCRIMINATOR SUBVARIETY	65	
CHAPTER	5 :	THE ABELIAN SUBVARIETY	73	
CHAPTER	6:	TRANSFER PRINCIPLES	75	
SUMMARY OF PART I				
PART II: ST	RUC	TURED ABELIAN VARIETIES		
CHAPTER	7:	STRONGLY SOLVABLE VARIETIES	95	
CHAPTER	8:	More transfer principles	99	
CHAPTER	9:	Consequences of the transfer principles	103	
CHAPTER	10:	Three interpretations	107	
CHAPTER	11:	FROM STRONGLY ABELIAN TO		
		ESSENTIALLY UNARY VARIETIES	129	
CHAPTER	12:	THE UNARY CASE	149	

CHAPTER 1	13:	THE DECOMPOSITION THEOREM	171
CHAPTER 1	l 4 :	Conclusion	193

PART III: THE DECOMPOSITION

CHAPTER 14	4: Conclusion	193
Bibliography		199
Notation		203
INDEX		207