Classics in Mathematics

Daniel W. Stroock
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Multidimensional Diffusion Processes

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Reprint of the 1997 Edition

Springer

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Originally published as Vol. 233 in the series
Grundlehren der mathematischen Wissenschaften

Mathematics Subject Classification (2000): 60J60, 28A65

Library of Congress Control Number: 2005934787

ISSN 1431-0821
ISBN 978-3-662-22201-0 ISBN 978-3-540-28999-9 (eBook)
DOI 10.1007/978-3-540-28999-9

Springer is a part of Springer Science+Business Media

springeronline.com

Originally published by Springer-Verlag Berlin Heidelberg New York in 2006

Production: LE-TEX Jelonek, Schmidt & Vöckler GbR, Leipzig
Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper 41/3142/YL - 5 4 3 2 1 0

Grundlehren der mathematischen Wissenschaften 233

A Series of Comprehensive Studies in Mathematics

Springer-Verlag Berlin Heidelberg GmbH

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Cataloging-in-Publication Data applied for

A catalog record for this book is available from the Library of Congress.

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;
detailed bibliographic data is available in the Internet at http://dnb.ddb.de

Mathematics Subject Classification (2000): 60J60, 28A65

ISSN 0072-7830

ISBN 978-3-662-22201-0 ISBN 978-3-540-28999-9 (eBook)
DOI 10.1007/978-3-540-28999-9

Springer-Verlag Berlin Heidelberg New York
a member of BertelsmannSpringer Science+Business Media GmbH

http://www.springer.de

Cover design: *design & production* GmbH, Heidelberg

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To our parents:
Katherine W. Stroock
Alan M. Stroock
S.R. Janaki
S.V. Ranga Ayyangar

Contents

Frequently Used Notation

I. Topological Notation. Let (X, ρ) be a separable metric space.

1) B° is the interior of $B \subseteq X$.
2) $\bar{B}$ is the closure of $B \subseteq X$.
3) ∂B is the boundary of $B \subseteq X$.
4) $\mathscr{B}_X$ is the Borel field of subsets of X.
5) $C_b(X)$ is the set of bounded continuous functions $f: X \to R$.
6) $B(X)$ is the set of bounded $\mathscr{B}_X$-measurable $f: X \to R$.
7) $U_\rho(X)$ is the set of bounded ρ-uniformly continuous $f: X \to R$.
8) $M(X)$ is the set of probability measures on $(X, \mathscr{B}_X)$.
9) $\|f\| = \sup_{x \in X} |f(x)|$ for $f \in B(X)$.

II. Special Notation for Euclidean Spaces

1) R^d is d-dimensional Euclidean space.
2) $|x| = (\sum_1^d x_j^2)^{1/2}$ for $x \in R^d$.
3) $B(x, r) = \{y \in R^d: |x - y| < r\}$.
4) $\langle x, y \rangle = \sum_1^d x_j y_j$ for $x, y \in R^d$.
5) $S^{d-1} = \{x \in R^d: |x| = 1\}$.
6) $\hat{C}(R^d) = \{f \in C_b(R^d): \lim_{|x| \to \infty} f(x) = 0\}$.
7) $C_0(\mathscr{G})$ is the set of $f \in C_b(\mathscr{G})$ having compact support.
8) $C_b^m(\mathscr{G})$ is the set of $f: \mathscr{G} \to R$ possessing bounded continuous derivatives of order up to and including m.
9) $C_b^\infty(\mathscr{G}) = \bigcap_{m=0}^\infty C_b^m(\mathscr{G})$.
10) $C^\infty(\mathscr{G})$ is the set of $f: \mathscr{G} \to R$ possessing continuous derivatives of all orders.
11) $C_0^\infty(\mathscr{G}) = C^\infty(\mathscr{G}) \cap C_0(\mathscr{G})$.
12) $C_b^{m,n}(\mathscr{G})$ for $\mathscr{G} \subseteq [0, \infty) \times R^d$ is the set of $f: \mathscr{G} \to R$ such that f has m bounded continuous time derivatives and bounded continuous spacial derivatives of order less than or equal to n.
13) $L^p(\mathscr{G})$, $1 \le p \le \infty$, is the usual L^p-space defined in terms of Lebesgue measure on $\mathscr{G}$.

14) $L^p_{\text{loc}}(\mathscr{G})$ is the set of $f: \mathscr{G} \to R$ (or $\mathbb{C}$) such that $f \in L^p(K)$ for all compact K in $\mathscr{G}$.

III. Path Spaces Notation

1) $C(I, R^d)$ for $I \subseteq [0, \infty)$ is the set of R^d-valued functions on I into R^d.
2) $\Omega_d(\Omega)$ (see p. 30).
3) $\mathscr{M}_t(\mathscr{M})$ (see p. 30).
4) $x(t, \omega)$ (see p. 30).

IV. Miscellaneous Notation

1) $a \wedge b$ is the smaller of the numbers $a, b \in R$.
2) $a \vee b$ is the larger of the numbers $a, b \in R$.
3) S_d is the set of symmetric non-negative definite $d \times d$ real matrices.
4) S_d^+ is the set of nondegenerate elements of S_d.
5) $\|A\|$, where A is a square matrix, and is the operator norm of A.
6) $\sigma(\mathscr{C})$, where $\mathscr{C}$ is a collection of subsets of X, and is the smallest σ-algebra over X containing $\mathscr{C}$.
7) $\sigma(\mathscr{F})$, where $\mathscr{F}$ is a set of functions on X into a measurable space, and is the smallest σ-algebra over X with respect to which every element of $\mathscr{F}$ is measurable.
8) $[\lambda]$, $\lambda \in R$, is the integral part of λ.
9) $\xi \sim I_d^s(a, b)$ (see p. 92).