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Weighted Hardy Spaces

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Preface

A considerable development of harmonic analysis in the last few years has been centered around a function space shown in a new light, the functions of bounded mean oscillation, and the weighted inequalities for classical operators. The new techniques introduced by C. Fefferman and E. Stein and B. Muckenhoupt are basic in these areas; for further details the reader may consult the monographs of García-Cuerva and Rubio de Francia [1985] and Torchinsky [1986]. It is our purpose here to further develop some of these results in the general setting of the weighted Hardy spaces, and to discuss some applications. The origin of these notes is the announcement in Strömberg and Torchinsky [1980], and the course given by the first author at Rutgers University in the academic year 1985-1986.

A word about the content of the notes. In Chapter I we introduce the notion of weighted measures in the general context of homogenous spaces; the results discussed here include the theory of A_p weights. Chapter II deals with the Jones decomposition of these weights including a novel feature, namely, the control of the doubling condition. In Chapter III we discuss the properties of the sharp maximal functions as well as those of the so-called local sharp maximal functions. This is also done in the context of homogeneous spaces, and the results proved include an extension of the John-Nirenberg inequality.

In Chapter IV we consider the functions defined on the upper-half space R^{n+1}_+ which are of interest to us, including the nontangential maximal function and the area function. Then, in Chapter V, we restrict our attention to a particular class of functions defined on R^{n+1}_+, namely, the extensions of a tempered distribution on R^n to the upper-half space R^{n+1}_+ by means of convolutions with the dilates of Schwartz functions. We study how the extension behaves with respect to different Schwartz functions, and an interesting result is the mean-value type inequality we show these extensions satisfy.

We are now ready to introduce the weighted Hardy spaces in Chapter VI. We also describe here some of their essential properties, such as the independence of the "norm" among others. In Chapter VII we construct a dense class of functions for these spaces of distributions; this is a delicate pursuit. Chapters VIII and IX lie at the heart of the matter: in Chapter VIII we construct the atomic decomposition for these spaces, and in Chapter IX we describe an extension of the Fefferman H^1 duality result by means of the so-called basic inequality.

In Chapters X, XI and XII we then discuss some applications. Chapter X contains the construction of the dual to the Hardy spaces, Chapter XI deals with the continuity of various singular integral and multiplier operators on these spaces, and, finally, in Chapter XII we show how the complex method of interpolation applies in these context. All in all, the essential ingredients of the theory of the weighted Hardy spaces is contained in these notes.

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