Undergraduate Texts in Mathematics

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James K. Strayer

Linear Programming and Its Applications

With 95 Illustrations

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James K. Strayer
Department of Mathematics
Lock Haven University
Lock Haven, PA 17745
USA

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Preface

Linear Programming and Its Applications is intended for a first course in linear programming, preferably in the sophomore or junior year of the typical undergraduate curriculum. The emphasis throughout the book is on linear programming skills via the algorithmic solution of small-scale problems, both in the general sense and in the specific applications where these problems naturally occur.

The book arose from lecture notes prepared during the years 1985–1987 while I was a graduate assistant in the Department of Mathematics at The Pennsylvania State University. I used a preliminary draft in a Methods of Management Science class in the spring semester of 1988 at Lock Haven University. Having been extensively tried and tested in the classroom at various stages of its development, the book reflects many modifications either suggested directly by students or deemed appropriate from responses by students in the classroom setting. My primary aim in writing the book was to address common errors and difficulties as clearly and effectively as I could.

The organization of the book attempts to achieve an orderly and natural progression of topics. The first part of the book deals with methods to solve general linear programming problems and discusses the theory of duality that connects these problems. Chapter 1 deals with solving linear programming problems geometrically; it is intended to constitute an introduction to the general linear programming problem through familiar geometrical concepts. At the same time, to motivate the study of a more effective procedure, the drawbacks of the geometric method are stressed. Chapter 2 develops the more effective procedure, the simplex algorithm of G. Dantzig. In this respect the book differs from several others in that it uses the condensed tableau of A.W. Tucker to record linear programming problems rather than the classical Dantzig tableau. The smaller size of the

Tucker tableau makes it much more amenable to both hand and computer calculations. Chapter 3 covers certain related problems that are not immediately solvable by the simplex algorithm, but, fortunately, can be easily converted to a form approachable by that method. (Such conversions are especially important in the second part of the book.) Chapter 4 concludes the first part of the book with a treatment of duality theory, a theory that establishes relationships between linear programming problems of maximization and minimization. The Tucker tableau approach makes an elegant presentation of this theory possible.

The second part of the book deals with several applications. These applications, besides being important in their own right, constitute introductions to important fields related to linear programming; the partial intention of this part of the book is the stimulation of the reader's interest in one or more of these fields. Chapter 5 introduces game theory. The methods applied to the games presented here are precisely those discussed in Chapters 2–4. Chapter 6 presents transportation and assignment problems, a large class of problems within operations research. Disadvantages of using the direct simplex algorithm in the solution of such problems are indicated and new algorithms related to it are developed. Finally, Chapter 7 introduces graph theory with a treatment of various network-flow problems. Direct and effective graph-theoretic linear programming algorithms are developed and duality in a specific network-flow problem is discussed in detail.

Appropriately for either a text or a reference book on linear programming, there are many examples and exercises. Virtually every definition is followed by several examples and every algorithm is illustrated in a step-by-step manner. The exercises range from easy computations to more difficult proofs and are chosen to elucidate and complement the exposition. To gain and reinforce comprehension of the material, the reader should attempt as many of these exercises as possible. The answers to all computational exercises appear in the back of the book; complete solutions to all exercises are in a supplementary solutions manual.

I tried to make *Linear Programming and Its Applications* approachable from as many levels (sophomore to graduate) and as many fields (mathematics, computer science, engineering, actuarial science, and economics) as posssible. The basic prerequisite is a knowledge of linear equations including the graphing of lines and planes as well as the solution (without matrices) of systems of simultaneous linear equations. Brief appendices on matrix algebra (for Chapters 2 and 4) and elementary probability (for Chapter 5) are included.

Each chapter of the book, except the introduction, is divided into sections (§'s). The symbol $m§n$ is to be read as "Chapter m, section n." The numbering of definitions, examples, and theorems proceeds sequentially throughout each chapter (i.e., Definition 1, Example 2, Definition 3, Theorem 4, etc.). The scheme is intended to make it easier to find any particular item. The numbering of mathematical statements and diagrams is similar. Any linear programming problem written in non-tableau form such as

$$\begin{aligned} &\text{Maximize} && P(x, y) = 30x + 50y \\ &\text{subject to} && 2x + y \leqq 8 \\ &&& x + 2y \leqq 10 \\ &&& x, y \geqq 0 \end{aligned}$$

is referred to by a single number as in

$$\begin{aligned} &\text{Maximize} && P(x, y) = 30x + 50y \\ &\text{subject to} && 2x + y \leqq 8 \\ &&& x + 2y \leqq 10 \\ &&& x, y \geqq 0. \end{aligned} \tag{1}$$

If individual statements in such a problem need to be referred to, decimal numbering will be used, as in

$$\begin{aligned} &\text{Maximize} && P(x, y) = 30x + 50y && (1.1) \\ &\text{subject to} && 2x + y \leqq 8 && (1.2) \\ &&& x + 2y \leqq 10 && (1.3) \\ &&& x, y \geqq 0. && (1.4) \end{aligned}$$

Throughout the book, the following standard notations are used:

Z: the set of integers
Q: the set of rational numbers
R: the set of real numbers
$\mathbf{R}^n$: n-dimensional real Euclidean space
$\forall$: "for all" or "for every."

The statement

$$\text{variable} \leftarrow \text{expression}$$

means "evaluate the expression and assign its value to the variable." Unless otherwise stated, all variables in this book represent real numbers.

I would like to express my sincere appreciation to the reviewers of the book as well as the fine staff of Springer-Verlag who assisted in the publication of the book. I must also thank the many students at Penn State University and Lock Haven University who shaped what the book was to become by offering comments, suggestions, and encouragement; the book is dedicated to them.

JAMES K. STRAYER

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