

Brian Straughan

Stability and Wave Motion in Porous Media

 Springer

Brian Straughan
Durham University
Department of Mathematical Sciences
Durham
UK
brian.straughan@durham.ac.uk

Editors

S.S. Antman
Department of Mathematics
and
Institute for Physical Science
and Technology
University of Maryland
College Park, MD 20742-4015
USA
ssa@math.umd.edu

J.E. Marsden
Control and Dynamical
Systems 107-81
California Institute of
Technology
Pasadena, CA 91125
USA
marsden@cds.caltech.edu

L. Sirovich
Laboratory of Applied
Mathematics
Department of
Biomathematical Sciences
Mount Sinai School
of Medicine
New York, NY 10029-6574
USA
chico@camelot.mssm.edu

ISBN: 978-0-387-76541-9

e-ISBN: 978-0-387-76543-3

DOI: 10.1007/978-0-387-76543-3

Library of Congress Control Number: 2008932580

Mathematics Subject Classification (2000): 76S05, 76E06, 76E05, 76E30, 74F10, 74F05, 74J30, 35B30, 35B35, 35B40, 35L60, 35Q35

© 2008 Springer Science+Business Media, LLC

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

Contents

Preface	vii
1 Introduction	1
1.1 Porous media	1
1.1.1 Applications, examples	1
1.1.2 Notation, definitions	6
1.1.3 Overview	9
1.2 The Darcy model	10
1.2.1 The Porous Medium Equation	11
1.3 The Forchheimer model	12
1.4 The Brinkman model	12
1.5 Anisotropic Darcy model	13
1.6 Equations for other fields	14
1.6.1 Temperature	14
1.6.2 Salt field	15
1.7 Boundary conditions	15
1.8 Elastic materials with voids	16
1.8.1 Nunziato-Cowin theory	16
1.8.2 Microstretch theory	17
1.9 Mixture theories	18
1.9.1 Eringen's theory	18
1.9.2 Bowen's theory	22

2	Structural Stability	27
2.1	Structural stability, Darcy model	27
2.1.1	Newton's law of cooling	28
2.1.2	<i>A priori</i> bound for T	30
2.2	Structural stability, Forchheimer model	31
2.2.1	Continuous dependence on b	32
2.2.2	Continuous dependence on c	34
2.2.3	Energy bounds	35
2.2.4	Brinkman-Forchheimer model	37
2.3	Forchheimer model, non-zero boundary conditions	37
2.3.1	A maximum principle for c	39
2.3.2	Continuous dependence on the viscosity	39
2.4	Brinkman model, non-zero boundary conditions	42
2.5	Convergence, non-zero boundary conditions	43
2.6	Continuous dependence, Vadasz coefficient	44
2.6.1	A maximum principle for T	45
2.6.2	Continuous dependence on α	46
2.7	Continuous dependence, Krishnamurti coefficient	48
2.7.1	An a priori bound for T	49
2.7.2	Continuous dependence	53
2.8	Continuous dependence, Dufour coefficient	55
2.8.1	Continuous dependence on γ	57
2.9	Initial - final value problems	69
2.10	The interface problem	72
2.11	Lower bounds on the blow-up time	76
2.12	Uniqueness in compressible porous flows	82
3	Spatial Decay	95
3.1	Spatial decay for the Darcy equations	95
3.1.1	Nonlinear temperature dependent density.	96
3.1.2	An appropriate "energy" function.	98
3.1.3	A data bound for $E(0, t)$	104
3.2	Spatial decay for the Brinkman equations	111
3.2.1	An estimate for $\text{grad } T$	112
3.2.2	An estimate for $\text{grad } \mathbf{u}$	114
3.3	Spatial decay for the Forchheimer equations	120
3.3.1	An estimate for $\text{grad } T$	125
3.3.2	An estimate for $E(0, t)$	127
3.3.3	An estimate for $u_i u_i$	129
3.3.4	Bounding ϕ_i	131
3.4	Spatial decay for a Krishnamurti model	132
3.4.1	Estimates for $T_{,i} T_{,i}$ and $C_{,i} C_{,i}$	134
3.4.2	An estimate for the $u_i u_i$ term	136
3.4.3	Integration of the H inequality	138

3.4.4	A bound for $H(0)$	138
3.4.5	Bound for $u_i u_i$ at $z = 0$	141
3.5	Spatial decay for a fluid-porous model	142
4	Convection in Porous Media	147
4.1	Equations for thermal convection in a porous medium	148
4.1.1	The Darcy equations	148
4.1.2	The Forchheimer equations	148
4.1.3	Darcy equations with anisotropic permeability	149
4.1.4	The Brinkman equations	150
4.2	Stability of thermal convection	150
4.2.1	The Bénard problem for the Darcy equations	151
4.2.2	Linear instability	152
4.2.3	Nonlinear stability	154
4.2.4	Variational solution to (4.28)	155
4.2.5	Bénard problem for the Forchheimer equations	158
4.2.6	Darcy equations with anisotropic permeability	159
4.2.7	Bénard problem for the Brinkman equations	163
4.3	Stability and symmetry	166
4.3.1	Symmetric operators	166
4.3.2	Heated and salted below	168
4.3.3	Symmetrization	170
4.3.4	Pointwise constraint	171
4.4	Thermal non-equilibrium	172
4.4.1	Thermal non-equilibrium model	172
4.4.2	Stability analysis	174
4.5	Resonant penetrative convection	177
4.5.1	Nonlinear density, heat source model	177
4.5.2	Basic equations	178
4.5.3	Linear instability analysis	180
4.5.4	Nonlinear stability analysis	181
4.5.5	Behaviour observed	182
4.6	Throughflow	183
4.6.1	Penetrative convection with throughflow	183
4.6.2	Forchheimer model with throughflow	184
4.6.3	Global nonlinear stability analysis	186
5	Stability of Other Porous Flows	193
5.1	Convection and flow with micro effects	193
5.1.1	Biological processes	193
5.1.2	Glia aggregation in the brain	194
5.1.3	Micropolar thermal convection	196
5.2	Porous flows with viscoelastic effects	198
5.2.1	Viscoelastic porous convection	198
5.2.2	Second grade fluids	200

5.2.3	Generalized second grade fluids	201
5.3	Storage of gases	202
5.3.1	Carbon dioxide storage	202
5.3.2	Hydrogen storage	204
5.4	Energy growth	205
5.4.1	Soil salinization	205
5.4.2	Other salinization theories	208
5.4.3	Time growth of parallel flows	210
5.4.4	Stability analysis for salinization	218
5.4.5	Transient growth in salinization	220
5.5	Turbulent convection	222
5.5.1	Turbulence in porous media	222
5.5.2	The background method	223
5.5.3	Selecting τ	225
5.6	Multiphase flow	227
5.6.1	Water-steam motion	227
5.6.2	Foodstuffs, emulsions	230
5.7	Unsaturated porous medium	231
5.7.1	Model equations	231
5.7.2	Stability of flow	232
5.7.3	Transient growth	233
5.8	Parallel flows	234
5.8.1	Poiseuille flow	234
5.8.2	Flow in a permeable conduit	236

6 Fluid - Porous Interface Problems 239

6.1	Models for thermal convection	239
6.1.1	Extended Navier-Stokes model	240
6.1.2	Nield (Darcy) model	241
6.1.3	Forchheimer model	243
6.1.4	Brinkman model	244
6.1.5	Nonlinear equation of state	244
6.1.6	Reacting layers	246
6.2	Surface tension	246
6.2.1	Basic solution	246
6.2.2	Perturbation equations	248
6.2.3	Perturbation boundary conditions	249
6.2.4	Numerical results	251
6.3	Porosity effects	253
6.3.1	Porosity variation	253
6.3.2	Numerical results	255
6.4	Melting ice, global warming	258
6.4.1	Three layer model	258
6.4.2	Under ice melt ponds	260
6.5	Crystal growth	262

6.6	Heat pipes	265
6.7	Poiseuille flow	267
6.7.1	Darcy model	267
6.7.2	Linearized perturbation equations	269
6.7.3	(Chang et al., 2006) results	271
6.7.4	Brinkman - Darcy model	272
6.7.5	Steady solution	273
6.7.6	Linearized perturbation equations	274
6.7.7	Numerical results	276
6.7.8	Forchheimer - Darcy model	276
6.7.9	Brinkman - Forchheimer / Darcy model	284
6.8	Acoustic waves, ocean bed	289
6.8.1	Basic equations	290
6.8.2	Linear waves in the Bowen theory	291
6.8.3	Boundary conditions	293
6.8.4	Amplitude behaviour	294
7	Elastic Materials with Voids	297
7.1	Acceleration waves in elastic materials	297
7.1.1	Bodies and their configurations	297
7.1.2	The deformation gradient tensor	298
7.1.3	Conservation of mass	298
7.1.4	The equations of nonlinear elasticity	298
7.1.5	Acceleration waves in one-dimension	300
7.1.6	Given strain energy and deformation	303
7.1.7	Acceleration waves in three dimensions	305
7.2	Acceleration waves, inclusion of voids	307
7.2.1	Porous media, voids, applications	307
7.2.2	Basic theory of elastic materials with voids	308
7.2.3	Thermodynamic restrictions	310
7.2.4	Acceleration waves in the isothermal case	312
7.3	Temperature rate effects	314
7.3.1	Voids and second sound	314
7.3.2	Thermodynamics and voids	316
7.3.3	Void-temperature acceleration waves	318
7.3.4	Amplitude behaviour	320
7.4	Temperature displacement effects	325
7.4.1	Voids and thermodynamics	325
7.4.2	De Cicco - Diaco theory	325
7.4.3	Acceleration waves	327
7.5	Voids and type III thermoelasticity	329
7.5.1	Thermodynamic theory	329
7.5.2	Linear theory	331
7.6	Acceleration waves, microstretch theory	332

8	Poroacoustic Waves	337
8.1	Poroacoustic acceleration waves	337
8.1.1	Equivalent fluid theory	337
8.1.2	Jordan - Darcy theory	339
8.1.3	Acceleration waves	340
8.1.4	Amplitude equation derivation	341
8.2	Temperature effects	344
8.2.1	Jordan-Darcy temperature model	344
8.2.2	Wavespeeds	345
8.2.3	Amplitude equation	346
8.3	Heat flux delay	349
8.3.1	Cattaneo poroacoustic theory	349
8.3.2	Thermodynamic justification	351
8.3.3	Acceleration waves	353
8.3.4	Amplitude derivation	356
8.3.5	Dual phase lag theory	358
8.4	Temperature rate effects	360
8.4.1	Green-Laws theory	360
8.4.2	Wavespeeds	362
8.4.3	Amplitude behaviour	364
8.5	Temperature displacement effects	366
8.5.1	Green-Naghdi thermodynamics	366
8.5.2	Acceleration waves	369
8.5.3	Wave amplitudes	371
8.6	Magnetic field effects	373
9	Numerical Solution of Eigenvalue Problems	375
9.1	The compound matrix method	375
9.1.1	The shooting method	375
9.1.2	A fourth order equation	376
9.1.3	The compound matrix method	377
9.1.4	Penetrative convection in a porous medium	379
9.2	The Chebyshev tau method	381
9.2.1	The D^2 Chebyshev tau method	381
9.2.2	Penetrative convection	384
9.2.3	Fluid overlying a porous layer	385
9.2.4	The D Chebyshev tau method	389
9.2.5	Natural variables	390
9.3	Legendre-Galerkin method	391
9.3.1	Fourth order system	391
9.3.2	Penetrative convection	395
9.3.3	Extension of the method	397
	References	399
	Index	433