

Chuanming Zong

# Strange Phenomena in Convex and Discrete Geometry

Edited by James J. Dudziak



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